Mini-Lecture 8.1

Right Triangles Trigonometry; Applications

Learning Objectives:

1. Find the Value of Trigonometric Functions of Acute Angles Using Right Triangles (p. 508)
2. Use the Complementary Angle Theorem (p. 510)
3. Solve Right Triangles (p. 510)
4. Solve Applied Problems (p. 511)

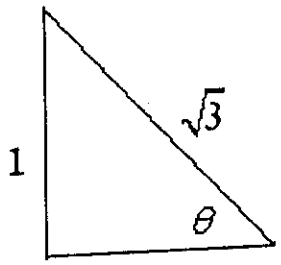
Examples:

1. In a right triangle, where C is the right angle, $a = 111.4\ cm$ and $b = 125.3\ cm$. Find c, A, and B.

2. A 500-foot cliff drops vertically into the ocean. If the angle of depression from the top of the cliff to a ship is $42°$, how far off shore, to the nearest foot, is the ship?

3. A security camera in a neighborhood bank is mounted on a wall 10 feet above the floor. What angle of depression should be used if the camera is to be directed to a spot 7 feet above the floor and 13 feet from the wall?

4. Find the six trigonometric functions of the angle θ in the figure below.



8.1 mini notes

$\sin\theta = \frac{\text{oppo}}{\text{hyp}}$ $\quad$ $\csc\theta = \frac{\text{hyp}}{\text{oppo}}$

$\cos\theta = \frac{\text{Adj}}{\text{hyp}}$ $\quad$ $\sec\theta = \frac{\text{hyp}}{\text{Adj}}$

$\tan\theta = \frac{\text{oppo}}{\text{adj}}$ $\quad$ $\cot\theta = \frac{\text{Adj}}{\text{oppo}}$

complementary Angles

$\sin 30^\circ = \cos 60^\circ$

$\tan 40^\circ = \cot 50^\circ$

$\sec 80^\circ = \csc 10^\circ$

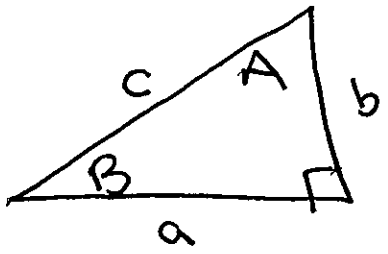
$c^2 = a^2 + b^2$

8.1 mini notes

① C is right angle

$a = 111.4$ cm, $b = 125.3$ cm

Find c, A, B



$a^2 + b^2 = c^2$

$111.4^2 + 125.3^2 = c^2$

$c = 167.7$

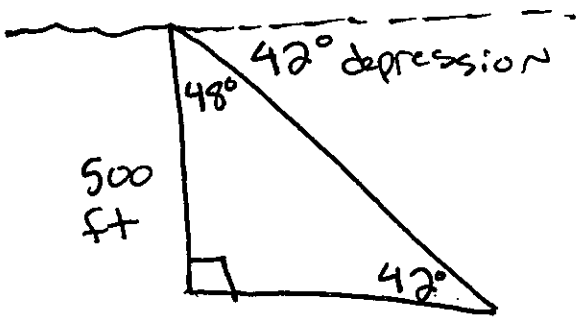
$\sin A = \frac{a}{c} = \frac{114.4}{167.7}$

$A = 41.6°$

$B = 48.4°$

8.1 mini lecture

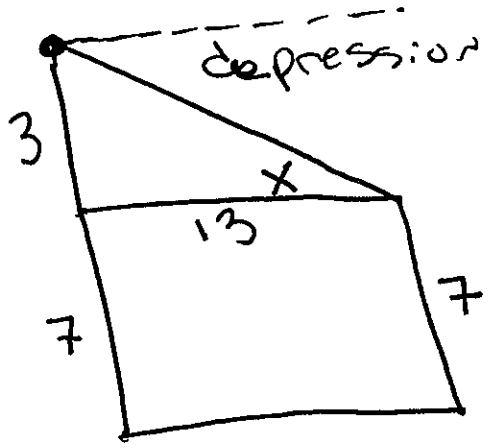
②



$$\tan 48^\circ = \frac{x}{500}$$

$$x = 555 \text{ ft}$$

③



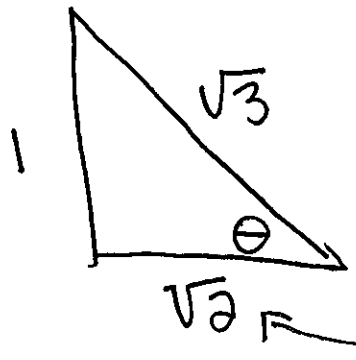
$$\tan x = \frac{3}{13}$$

$$\tan^{-1}\left(\frac{3}{13}\right) = x$$

$$x = 13^\circ$$

8.1 mini notes

(4)



$1^2 + x^2 = (\sqrt{3})^2$

$x^2 = 3 - 1$

$x = \sqrt{2}$

$\sin\theta = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

$\cos\theta = \frac{\sqrt{2}}{\sqrt{3}} = \boxed{\frac{\sqrt{6}}{3}}$

$\tan\theta = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

$\cot\theta = \frac{\sqrt{2}}{1} = \boxed{\sqrt{2}}$

$\csc\theta = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$

$\sec\theta = \frac{\sqrt{3}}{\sqrt{2}} = \boxed{\frac{\sqrt{6}}{2}}$

Mini-Lecture 8.2
The Law of Sines

Learning Objectives:

1. Solve SAA or ASA Triangles (p. 521)
2. Solve SSA Triangles (p. 522)
3. Solve Applied Problems (p. 525)

Examples:

1. $A = 37^\circ,\ C = 75^\circ,\ a = 11$

2. $a = 44,\ b = 62,\ A = 42^\circ$

3. $b = 15,\ c = 25,\ B = 63^\circ$

4. Two fire-lookout stations are 30 miles apart, with Station B directly west of Station A. Both stations spot a fire on a mountain to the south. The bearing from station A to the fire is $N32^\circ W$. The bearing from station B to the fire is $N40^\circ E$. How far, to the nearest mile, is the fire from station B?

8.2 mini lecture notes

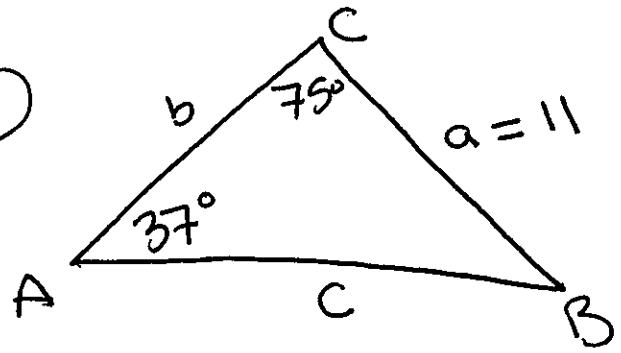
Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$A + B + C = 180^\circ$$

8.2 mini notes

(1)



Given

$A = 37°$	$a = 11$
$B = ?$	$b = ?$
$C = 75°$	$c = ?$

$$\frac{\sin 37}{11} = \frac{\sin 75}{C}$$

$C = 17.66$

$B = 68°$

$180 - 37 - 75$

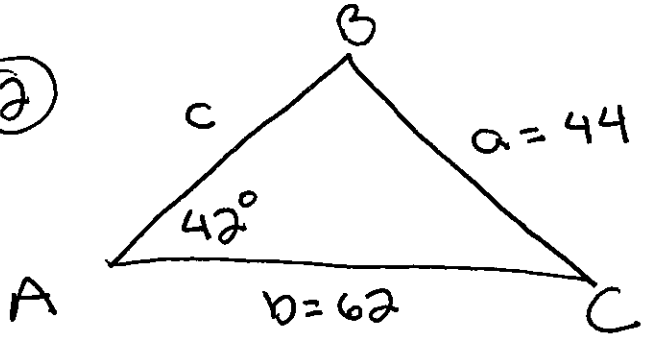
$$\frac{\sin 37}{11} = \frac{\sin 68}{b}$$

$b = 16.95$

* angles are locked in *

8.2 mini notes

②



$\frac{\sin 42}{44} = \frac{\sin B}{62}$ $\quad B = 70.46^\circ$

$180 - 42 - 70.46 = C$ $\quad C = 67.54^\circ$

$\frac{\sin 42}{44} = \frac{\sin 67.54}{C}$ $\quad c = 60.8$

* Since one angle given, check for other solutions *

$180 - 70.46 =$ $109.54^\circ = B$

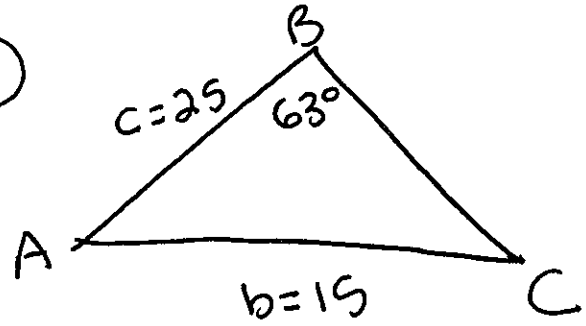
$180 - 109.54 - 42 =$ $28.46^\circ = C$

$\frac{\sin 42}{44} = \frac{\sin 109.54}{62} = \frac{\sin 28.46}{C}$

$c = 31.35$

8.2 mini notes

③



$$\frac{\sin 63}{15} = \frac{\sin C}{25}$$

$\sin C = 1.485$

$\sin^{-1}(1.485) = C$

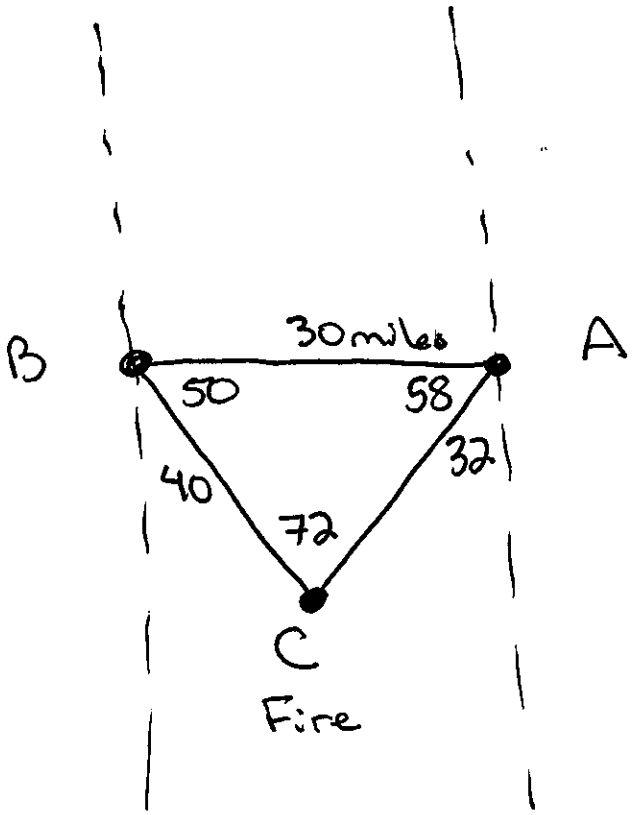
*not defined

$[-1, 1]$

no solution

8.2 mini notes

(4)



$$\frac{\sin 72}{30} = \frac{\sin 58}{\overline{BC}}$$

$\overline{BC} = 26.75$

$\overline{BC} = 27$ miles

Mini-Lecture 8.3

The Law of Cosines

Learning Objectives:

1. Solve SAS Triangles (p. 532)
2. Solve SSS Triangles (p. 533)
3. Solved Applied Problems (p. 533)

Examples:

Solve the oblique triangles.

1. $a = 26,\ b = 32,\ C = 110^\circ$

2. $a = 32,\ b = 46,\ c = 69$

3. $b = 6,\ c = 3,\ A = 70^\circ$

4. A plane leaves airport A and travels 690 miles to airport B on a bearing of N 42° E. The plane later leaves airport B and travels to airport C 525 miles away on a bearing of S 81° E. Find the distance from airport A to airport C to the nearest mile.

8.3 notes mini lecture

Law of Cosines (SAS) or (SSS)

$$c^2 = a^2 + b^2 - 2ab\cos C$$

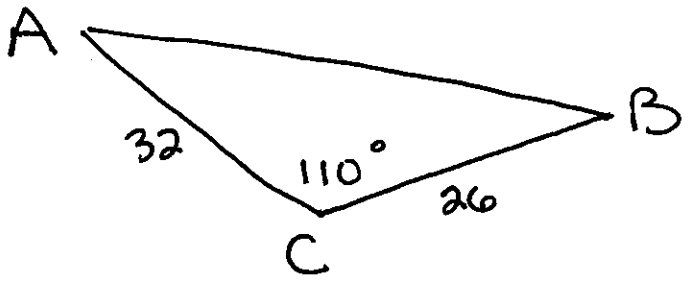
$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

8.3 mini notes

①

$a = 26, b = 32, C = 110^\circ$



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 26^2 + 32^2 - 2(26)(32)\cos 110^\circ$

$C = 47.6$

$\frac{\sin 110}{47.6} = \frac{\sin A}{26}$ $A = 30.9^\circ$

$\frac{\sin 110}{47.6} = \frac{\sin B}{32}$ $B = 39.1^\circ$

8.3 min: notes

② $a = 32$, $b = 46$, $C = 69$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$69^2 = 32^2 + 46^2 - 2(32)(46)\cos C$$

$$C = 123.4^\circ$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$32^2 = 46^2 + 69^2 - 2(46)(69)\cos A$$

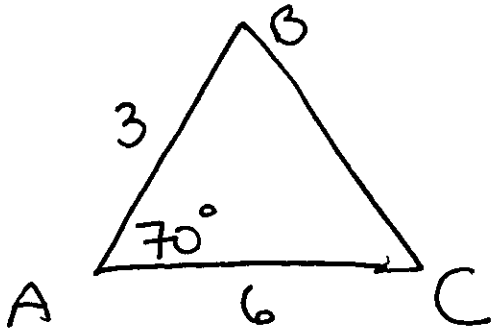
$$A = 22.8^\circ$$

$$180 - 123.4 - 22.8 = B$$

$$B = 33.8^\circ$$

8.3 mini notes

③ $b = 6$, $c = 3$, $A = 70^\circ$



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 6^2 + 3^2 - 2(6)(3)\cos 70$

$a = 5.7$

* depending on rounding, answer may be different

$\frac{\sin 70}{5.7} = \frac{\sin B}{6}$

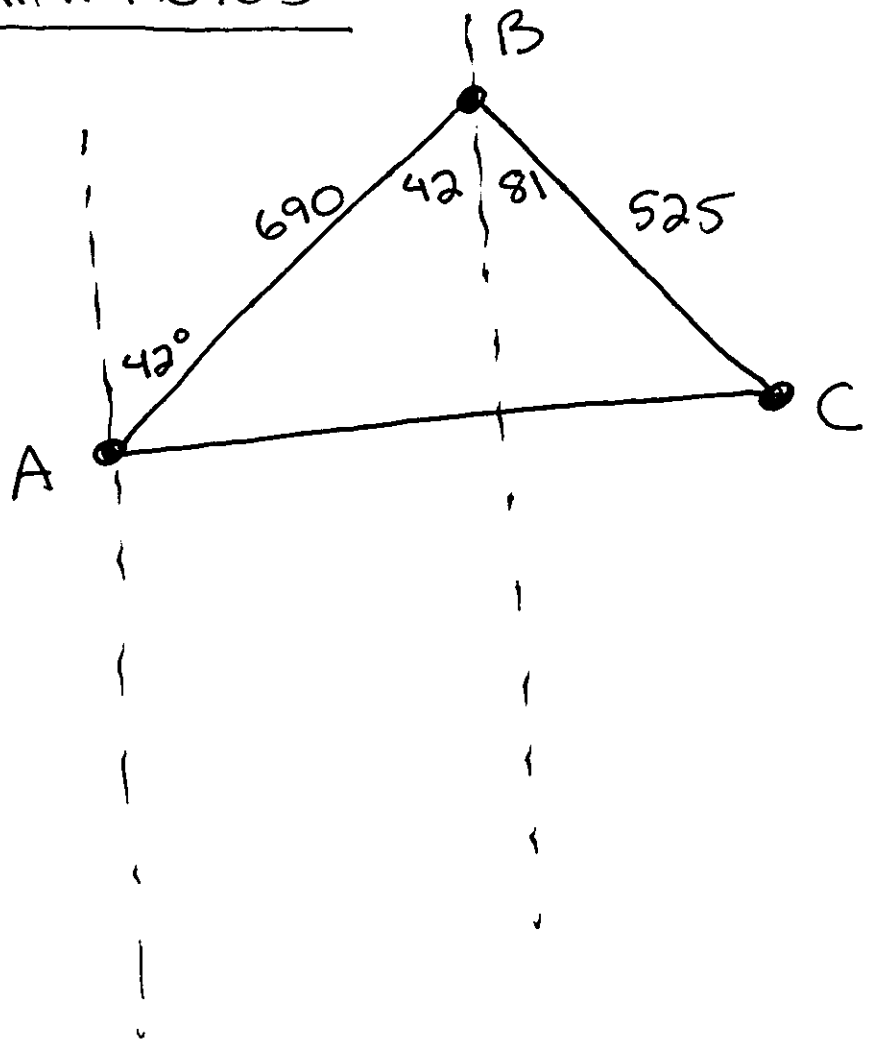
$B = 81.6^\circ$

$180 - 81.6 - 70 = C$

$C = 28.4^\circ$

8.3 mini notes

(4)



$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 525^2 + 690^2 - 2(525)(690)\cos 123$

$b = 1070.7$

Mini-Lecture 8.4

Area of a Triangle

Learning Objectives:

1. Find the Areas of SAS Triangles (p. 538)
2. Find the Area of SSS Triangles (p. 539)

Examples:

1. Find the area of a triangle with sides: $a = 82,\ b = 104,\ c = 67$. Measurements are in feet.

2. Find the area of the triangle having the given measurements.

   $B = 112^\circ$, $a = 7$ meters, $c = 9$ meters

3. Find the area of a triangle with sides: $a = 12,\ b = 14,\ c = 19$. Measurements are in feet.

4. Find the area of the segment of a circle whose radius is 8 inches, formed by a central angle of 50°.

8.4 mini notes

Area of Triangle $K = \frac{1}{2}bh$

SAS Δ

$$K = \frac{1}{2}ab\sin C$$

$$K = \frac{1}{2}bc\sin A$$

$$K = \frac{1}{2}ac\sin B$$

SSS Δ

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a+b+c)$

Heron's Formula

8.4 mini notes

① $a = 82, b = 104, c = 67$

$$s = \frac{1}{2}(82 + 104 + 67) = 126.5$$

$$K = \sqrt{126.5(126.5 - 82)(126.5 - 104)(126.5 - 67)}$$

$$K = 2745.2 \text{ ft}^2$$

② $B = 112^\circ, a = 7, c = 9$

$$K = \frac{1}{2} ac \sin B$$

$$K = \frac{1}{2}(7)(9)(\sin 112)$$

$$K = 29.2 \text{ m}^2$$

8.4 mini lecture notes

(3) $a = 12, b = 14, c = 19$

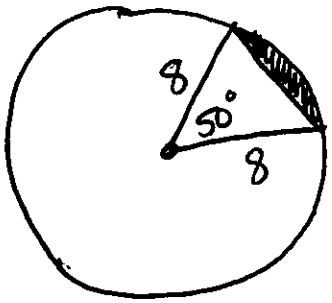
$$s = \frac{1}{2}(12+14+19) = 22.5$$

$$K = \sqrt{22.5(10.5)(8.5)(3.5)}$$

$$\boxed{K = 83.8 \text{ ft}^2}$$

8.4 min: notes

(4)



* θ must be radians

$$\text{Sector Area} = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2}(8^2)(50)\left(\pi/180\right)$$

$$= \frac{80\pi}{9} \text{ ft}^2$$

$$\text{Triangle} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2}(8)(8)(\sin 50)$$

$$= 24.513\,42218$$

$$\text{Segment} = \frac{80\pi}{9} - 24.513\,42218$$

$$= \boxed{3.4 \text{ ft}^2}$$

Mini-Lecture 8.5

Simple Harmonic Motion: Damped Motion; Combining Waves

Learning Objectives:

1. Build a Model for an Object in Simple Harmonic Motion (p. 544)
2. Analyze Simple Harmonic Motion (p. 546)
3. Analyze an Object in Damped Motion (p. 546)
4. Graph the Sum of Two Functions (p. 548)

Examples:

1. Suppose that an object attached to a coiled spring is pulled down a distance of 6 inches from its rest position and then released. If the time for one oscillation is 4 seconds, develop a model that relates the displacement d of an object from its rest position after time t (in seconds). Assume no friction.

2. An object moves in simple harmonic motion described by $d = -3\sin 4\pi t$, where t is measured in seconds and d in measured in inches. Find the maximum displacement, the frequency, and the time required for one cycle.

3. The distance d in meters of the bob of a pendulum of mass m (in kilograms) from its rest position at time t (in seconds) is given. The bob is released from the left of its rest position, which represents a negative direction. What is the initial displacement of the bob? Graph the motion using a graphing utility. What is the displacement of the bob at the start of the second oscillation?

$$d = -25e^{-0.5t/16}\cos\left(\sqrt{\left(\frac{2\pi}{3}\right)^2 - \frac{0.25}{256}}t\right)$$

4. Use the method of adding y-coordinates to graph the function.

$$y = \sin x - \sin(2x)$$

8.5 mini notes

Simple harmonic motion

$$d = a\cos(\omega t) \quad \text{or} \quad d = a\sin(\omega t)$$

Amplitude $= |a|$

$$\text{period} = \frac{2\pi}{\omega}$$

$$\text{frequency} = \frac{\omega}{2\pi}$$

Damped motion

$$d(t) = ae^{-bt/(2m)}\cos\left(\sqrt{\omega^2 - \frac{b^2}{4m^2}}\ t\right)$$

$|a|$ = displacement at $t = 0$

$$\text{period} = \frac{2\pi}{\omega}$$

8.5 mini notes

① $d = a\cos(\omega t)$

$d = -6$ when $t = 0$

so cos is easier to use

$a = -6$

$\text{period} = \frac{2\pi}{\omega}$ $\quad 4 = \frac{2\pi}{\omega}$ $\quad \omega = \pi/2$

$$d = -6\cos\left(\frac{\pi}{2}t\right)$$

8.5 mini notes

② $d = -3\sin(4\pi t)$

max displacement is amplitude

$= \boxed{3 \text{ inches}}$

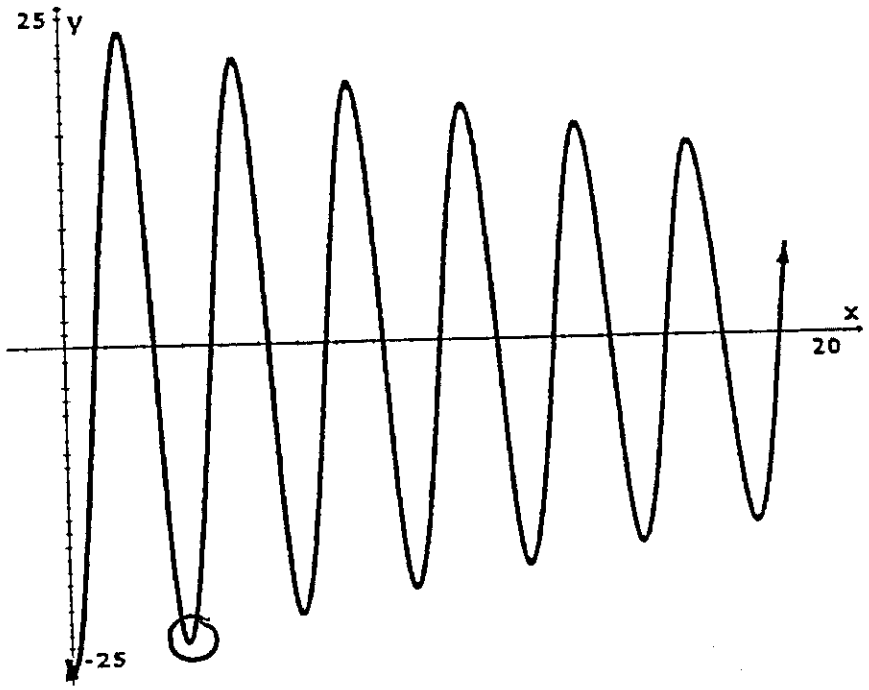
$$\text{frequency} = \frac{\omega}{2\pi} = \frac{4\pi}{2\pi} = \boxed{2 \text{ cycles per second}}$$

$$\text{one cycle} = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = \boxed{\frac{1}{2} \text{ second}}$$

(3) $d = -25e^{-.5t/16}\cos\left(\sqrt{\left(\frac{2\pi}{3}\right)^2 - \frac{.25}{256}}\ t\right)$

* type in calculator and graph

Initial displacement = -25 meters



(2.99, -22.77)

22.77 leftward

8.5 min lecture

④ $y = \sin x - \sin 2x$

	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$	2π
$\sin x$	-1	0	1	0	-1	0
$\sin 2x$	0	0	0	0	0	0
$\sin x - \sin 2x$	-1	0	1	0	-1	0

$(-\pi/2, -1)$

$(0, 0)$

$(\pi/2, 1)$

$(\pi, 0)$

$(3\pi/2, -1)$

$(2\pi, 0)$