Student: ____________ **Date:** ____________	**Instructor:** Joe Betters **Course:** Pre-Calculus Pre AP (Master Course)	**Assignment:** Chapter 7 Review

1. Find the inverse function f^{-1} of the function f. Find the range of f and the domain and range of f^{-1}.

$$f(x) = -\tan(x+7) - 2; \ -7 - \frac{\pi}{2} < x < \frac{\pi}{2} - 7$$

Find the inverse, f^{-1}, of f.

$f^{-1}(x) =$ ______
(Use integers or fractions for any numbers in the expression.)

The range of f(x) is ______.
(Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)

The domain of $f^{-1}(x)$ is ______.
(Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)

The range of $f^{-1}(x)$ is ______.
(Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)

ID: 7.1.57

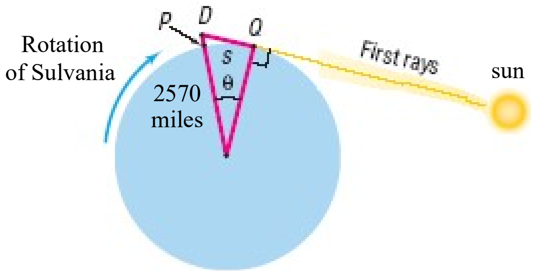
2. Find the exact solution of the equation.

$$9\cos^{-1}x - 2\pi = \cos^{-1}x$$

The solution set is { ______ }.
(Simplify your answer, including any radicals. Type an exact answer, using radicals as needed.)

ID: 7.1.67

3. On the planet Sulvania, a mountain with an elevation of 1180 feet is the highest peak on the east coast of a land mass. The summit of the mountain is the first place on the land mass to be lit by the rays of the sun. How much sooner do the first rays of the sun touch the summit than the base of the mountain below, at sea level? Assume that one rotation of Sulvania takes 24 hours and that the radius of Sulvania is 2570 miles.



The first rays of the sun touch the summit about ______ minutes sooner than the base of the mountain below, at sea level.
(Round to two decimal places as needed.)

ID: 7.1.75

4. Write the trigonometric expression as an algebraic expression in u.

$\mathbf{sin}\left(\mathbf{sec}^{-1} u\right)$

$\mathbf{sin}\left(\mathbf{sec}^{-1} u\right) =$ ______ (Type an exact answer, using radicals as needed.)

ID: 7.2.63

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5. Let f(x) = **sin** x, g(x) = **cos** x, and h(x) = **tan** x. Find the exact value of the composite function.

$g\left(h^{-1}\left(\frac{24}{7}\right)\right)$

$g\left(h^{-1}\left(\frac{24}{7}\right)\right) =$ ______

(Simplify your answer, including any radicals. Type an exact answer, using radicals as needed. Type an exact answer, using π as needed. Use integers or fractions for any numbers in the expression.)

ID: 7.2.73

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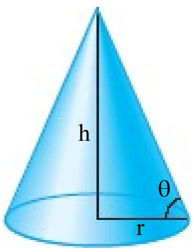
6. When granular materials fall, they form conical piles. The angle of slope, measured from the horizontal, at which the material comes to rest is called the angle of repose. The angle of repose is related to the height h and base radius r of the conical pile by the equation $\theta = \mathbf{cot}^{-1}\frac{r}{h}$. When certain granular materials are stored in a pile 11 feet high, the diameter of the base of the pile is 45 feet.



(a) Find the angle of repose for these granular materials.

θ = ______ ° (Round to two decimal places as needed.)

(b) What is the base diameter of a pile that is 8 feet high?

diameter = ______ feet (Round to two decimal places as needed.)

(c) What is the height of a pile that has a base diameter of approximately 104 feet?

height = ______ feet (Round to two decimal places as needed.)

ID: 7.2.79

7. Solve the equation. Give a general formula for all the solutions. List all the solutions for k = 0, 1, and 2.

$$\sin(2\theta) = -\frac{1}{2}$$

Write the general form for all the solutions to $\sin(2\theta) = -\frac{1}{2}$ based on the smaller angle.

$\theta =$ ______

(Simplify your answer. Type an exact answer, using π as needed. Use integers or fractions for any numbers in the expression. Type an expression using k as the variable.)

Write the general form for all the solutions to $\sin(2\theta) = -\frac{1}{2}$ based on the larger angle.

$\theta =$ ______

(Simplify your answer. Type an exact answer, using π as needed. Use integers or fractions for any numbers in the expression. Type an expression using k as the variable.)

List the solutions for k = 0, 1, and 2.

$\theta =$ ______

(Simplify your answer. Type an exact answer, using π as needed. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

ID: 7.3.41

8. Solve the equation $x - 6\cos x = 0$ by using a graphing utility.

What is the solution? Select the correct choice and fill in any answer boxes in your choice below.

- **A.** The solution set is { ______ }.
  (Round to the nearest two decimal places as needed. Use a comma to separate answers as needed.)
- **B.** There is no solution.

ID: 7.3.81

9. What are the zeros of $f(x) = 4\sin^2 x - 3$ on the interval $[0,2\pi]$?

The zeros are ______.

(Type an exact answer in terms of π. Use a comma to separate answers as needed.)

ID: 7.3.93

10. Establish the identity.

$\sec\theta \cdot \sin\theta = \tan\theta$

Write the left side in terms of sine and cosine.

$\boxed{} \cdot \sin\theta$

Write the result from the previous step as a single fraction.

$\boxed{}$ (Do not simplify.)

The fraction from the previous step then simplifies to **tan** θ using what?

- **A.** Pythagorean Identity
- **B.** Cancellation Property
- **C.** Quotient Identity
- **D.** Even-Odd Identity
- **E.** Reciprocal Identity

ID: 7.4.19

11. Establish the identity.

$\left(1+\tan^2\theta\right)\cos^2\theta = 1$

Rewrite the left side expression by expanding the product.

$\boxed{}$ (Do not simplify.)

Apply the appropriate quotient identity and/or the appropriate reciprocal identity to the expression in the previous step.

$\cos^2\theta + \left(\boxed{}\right)\cos^2\theta$

Simplify the expression from the previous step by canceling the common factors.

$\cos^2\theta + \boxed{}$

The expression from the previous step then simplifies to 1 using what?

- **A.** Even-Odd Identity
- **B.** Pythagorean Identity
- **C.** Cancellation Property
- **D.** Quotient Identity
- **E.** Reciprocal Identity

ID: 7.4.31

12. Establish the identity.

$$\frac{\left(1-2\sin^2\theta\right)^2}{\cos^4\theta-\sin^4\theta}=1-2\sin^2\theta$$

Use a Pythagorean identity to rewrite the numerator in terms of $\cos\theta$ and $\sin\theta$ and factor the denominator into two factors by factoring the difference of two squares.

After cancelling common factors from the previous fraction, use a Pythagorean identity to rewrite the denominator. Write the new denominator below.

The entire expression can now be rewritten as $1-2\sin^2\theta$ using what?

- **A.** Quotient Identity
- **B.** Even-Odd Identity
- **C.** Pythagorean Identity
- **D.** Cancellation Property
- **E.** Reciprocal Identity

ID: 7.4.85

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13. Find the exact value of each of the following under the given conditions:

$$\tan\alpha=-\frac{3}{4},\ \frac{\pi}{2}<\alpha<\pi;\ \cos\beta=\frac{5}{8},\ 0<\beta<\frac{\pi}{2}$$

(a) $\sin(\alpha+\beta)$ (b) $\cos(\alpha+\beta)$ (c) $\sin(\alpha-\beta)$ (d) $\tan(\alpha-\beta)$

(a) $\sin(\alpha+\beta)=$ ______
(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

(b) $\cos(\alpha+\beta)=$ ______
(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

(c) $\sin(\alpha-\beta)=$ ______
(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

(d) $\tan(\alpha-\beta)=$ ______
(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

ID: 7.5.35

14. Establish the identity.

$$\tan(\alpha - \beta) = \frac{\cot\beta - \cot\alpha}{\cot\alpha\cot\beta + 1}$$

Choose the sequence of steps below that verifies the identity.

A.

$$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta} = \frac{\dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\sin\alpha\sin\beta}}{\dfrac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\sin\beta}}$$

$$= \frac{\dfrac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta} - \dfrac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}}{\dfrac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} + \dfrac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}} = \frac{\cot\beta - \cot\alpha}{\cot\alpha\cot\beta + 1}$$

B.

$$\tan(\alpha-\beta) = \frac{\cos(\alpha-\beta)}{\sin(\alpha-\beta)} = \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\sin\alpha\cos\beta + \cos\alpha\sin\beta} = \frac{\dfrac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\sin\alpha\sin\beta}}{\dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\sin\beta}}$$

$$= \frac{\dfrac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} - \dfrac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}}{\dfrac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta} + \dfrac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}} = \frac{\cot\beta - \cot\alpha}{\cot\alpha\cot\beta + 1}$$

C.

$$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta} = \frac{\dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$= \frac{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} - \dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}} = \frac{\cot\beta - \cot\alpha}{\cot\alpha\cot\beta + 1}$$

ID: 7.5.65

15. Find the exact value of the expression.

$$\sin\left(\sin^{-1}\frac{12}{13} + \tan^{-1}\frac{3}{4}\right)$$

$\sin\left(\sin^{-1}\frac{12}{13} + \tan^{-1}\frac{3}{4}\right) =$ ________

(Type an exact answer in simplified form.)

ID: 7.5.79

16. Write the trigonometric expression as an algebraic expression containing u and v. Give the restrictions required on u and v.

$\cos\left(\tan^{-1}u - \sin^{-1}v\right)$

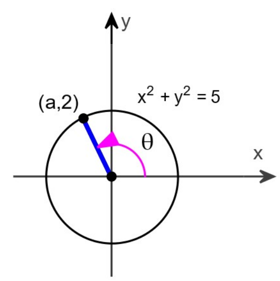
Choose the correct answer below.

- A. $\dfrac{\sqrt{1-v^2}+uv}{\sqrt{1+u^2}}, -\dfrac{\pi}{2} \le u \le \dfrac{\pi}{2}, -\dfrac{\pi}{2} \le v \le \dfrac{\pi}{2}$
- B. $\dfrac{\sqrt{1-v^2}+uv}{\sqrt{1+u^2}}, -\infty < u < \infty, -1 \le v \le 1$
- C. $\dfrac{v+u\sqrt{1-v^2}}{\sqrt{1+u^2}}, -\infty < u < \infty, -1 \le v \le 1$

ID: 7.5.87

17. Use the figure to evaluate the function given that $f(x) = \sin x$.

$f(2\theta)$



$f(2\theta) =$ ______ (Simplify your answer.)

ID: 7.6.29

18. Establish the identity. $\tan(18\theta) = \dfrac{3\tan(6\theta) - \tan^3(6\theta)}{1 - 3\tan^2(6\theta)}$

Choose the sequence of steps below that verifies the identity.

- A. $\tan(12\theta + 6\theta) = \dfrac{\tan(12\theta) + \tan(6\theta)}{1 - \tan(12\theta)\tan(6\theta)} = \dfrac{\dfrac{2\tan(6\theta)}{1-\tan^2(6\theta)} + \tan(6\theta)}{1 - \dfrac{2\tan(6\theta)}{1-\tan^2(6\theta)}\tan(6\theta)} = \dfrac{3\tan(6\theta) - \tan^3(6\theta)}{1 - 3\tan^2(6\theta)}$
- B. $\tan(24\theta - 6\theta) = \dfrac{\tan(12\theta) + \tan(6\theta)}{1 - \tan(12\theta)\tan(6\theta)} = \dfrac{\dfrac{2\tan(6\theta)}{1+\tan^2(6\theta)} + \tan(6\theta)}{1 - \dfrac{2\tan(6\theta)}{1+\tan^2(6\theta)}\tan(6\theta)} = \dfrac{3\tan(6\theta) - \tan^3(6\theta)}{1 - 3\tan^2(6\theta)}$
- C. $\tan(24\theta - 6\theta) = \dfrac{\tan(24\theta) + \tan(6\theta)}{1 - \tan(24\theta)\tan(6\theta)} = \dfrac{\dfrac{2\tan(12\theta)}{1-\tan^2(12\theta)} - \tan(6\theta)}{1 + \dfrac{2\tan(12\theta)}{1-\tan^2(12\theta)}\tan(6\theta)} = \dfrac{3\tan(6\theta) - \tan^3(6\theta)}{1 - 3\tan^2(6\theta)}$

ID: 7.6.65

19. Solve the equation on the interval $0 \le \theta < 2\pi$.

$$3\cos(2\theta) = -3\cos\theta$$

Select the correct choice below and fill in any answer boxes in your choice.

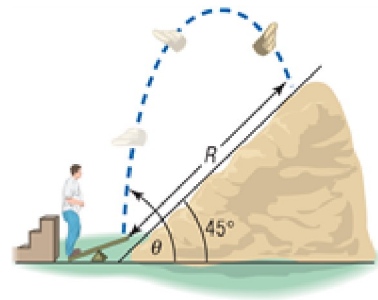
- **A.** $\theta = \{ \_\_\_\_\_ \}$ radians
  (Simplify your answer. Type an exact answer, using π as needed. Type your answer in radians. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)
- **B.** There is no solution on this interval.

ID: 7.6.71

20.

An object is propelled upward at an angle θ, $45^\circ < \theta < 90^\circ$, to the horizontal with an initial velocity of v_0 feet per second from the base of a plane that makes an angle of 45° with the horizontal. See the illustration. If air resistance is ignored, the distance R that it travels up the inclined plane is given by the function

$$R(\theta) = \frac{v_0^2\sqrt{2}}{16}\cos\theta(\sin\theta - \cos\theta).$$



(a) Using double-angle identities, determine which of the following equations is equivalent to the one given in the problem statement.

- A. $R(\theta) = \frac{v_0^2\sqrt{2}}{32}[\sin(2\theta) - \cos(2\theta) - 1]$
- B. $R(\theta) = \frac{v_0^2\sqrt{2}}{32}[\tan(2\theta) - \cos(2\theta) - 1]$
- C. $R(\theta) = \frac{v_0^2\sqrt{2}}{32}[\cos(2\theta) - \sin(2\theta) - 1]$

(b) The angle θ that maximizes R, satisfies this equation $\sin(2\theta) + \cos(2\theta) = 0$. Solve this equation for θ.

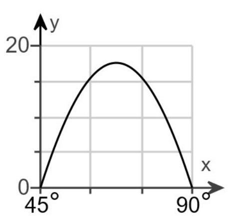
_____ °

(c) What is the maximum distance R if $v_0 = 31$ feet per second?

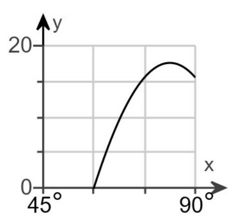
_____ feet (Round to the nearest hundredth as needed.)

(d) Graph $R = R(\theta)$, $45^\circ < \theta < 90^\circ$, and find the angle θ that maximizes the distance R. Also find the maximum distance. Use $v_0 = 31$ feet per second. Compare the results with the answers found earlier. Choose the correct graph below.

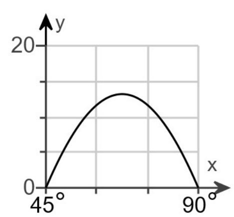
A.



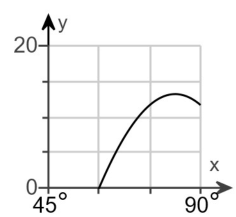
B.



C.



D.



The graph indicates that the angle θ that maximizes the distance R is _____ ° and the maximum distance is _____ feet. (Round to the nearest hundredth as needed.)

These results (1) _____ the answers in parts (b) and (c).

(1) ○ do not match
○ match

ID: 7.6.97

21. Express the given sum or difference as a product of sines and/or cosines.

$$\cos\frac{3\theta}{2} - \cos\frac{13\theta}{2}$$

$\cos\frac{3\theta}{2} - \cos\frac{13\theta}{2} =$ ______

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

ID: 7.7.23

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22. Establish the identity.

$$\frac{\cos y + \cos x}{\cos y - \cos x} = -\cot\frac{y+x}{2}\cot\frac{y-x}{2}$$

Choose the correct sequence of steps to establish the identity.

A. $\frac{\cos y + \cos x}{\cos y - \cos x} = \frac{2\cos\frac{y+x}{2}\cos\frac{y-x}{2}}{-2\sin\frac{y+x}{2}\sin\frac{y-x}{2}} = -\cot\frac{y+x}{2}\cot\frac{y-x}{2}$

B. $\frac{\cos y + \cos x}{\cos y - \cos x} = \frac{-2\sin\frac{y+x}{2}\sin\frac{y-x}{2}}{2\cos\frac{y+x}{2}\cos\frac{y-x}{2}} = -\cot\frac{y+x}{2}\cot\frac{y-x}{2}$

C. $\frac{\cos y + \cos x}{\cos y - \cos x} = \frac{-2\sin\frac{y-x}{2}\cos\frac{y+x}{2}}{2\sin\frac{y+x}{2}\cos\frac{y-x}{2}} = -\cot\frac{y+x}{2}\cot\frac{y-x}{2}$

ID: 7.7.37

23. Derive the Product-to-Sum formula $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$.

First, rewrite the term $\sin(\alpha+\beta)$, using the Sum Formula for the sine function.

$\sin(\alpha+\beta) =$ ______

Now, rewrite the term $\sin(\alpha-\beta)$, using the Difference Formula for the sine function.

$\sin(\alpha-\beta) =$ ______

Next, substitute the resulting expressions into the expression $\frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$, for $\sin(\alpha+\beta)$ and $\sin(\alpha-\beta)$ respectively, and then simplify.

$\frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)] = \frac{1}{2}[\ \_\_\_\_\_\_\ ]$ (Simplify your answer.)

$\frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)] =$ ______ (Simplify your answer.)

Therefore, the resulting formula is shown below.

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

ID: 7.7.53

1. $-\tan^{-1}(x+2)-7$

   $(-\infty,\infty)$

   $(-\infty,\infty)$

   $\left(-7-\frac{\pi}{2},\frac{\pi}{2}-7\right)$

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2. $\frac{\sqrt{2}}{2}$

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3. 3.02

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4. $\sqrt{1-\frac{1}{u^2}}$

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5. $\frac{7}{25}$

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6. 26.05

   32.74

   25.42

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7. $\frac{7\pi}{12}+k\pi$

   $\frac{11\pi}{12}+k\pi$

   $\frac{7\pi}{12},\frac{11\pi}{12},\frac{19\pi}{12},\frac{23\pi}{12},\frac{31\pi}{12},\frac{35\pi}{12}$

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8. A. The solution set is $\{$**−3.99, −1.89, 1.34**$\}$.
   (Round to the nearest two decimal places as needed. Use a comma to separate answers as needed.)

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9. $\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}$

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10. $\frac{1}{\cos\theta}$

$\frac{\sin\theta}{\cos\theta}$

C. Quotient Identity

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11. $\cos^2\theta + \tan^2\theta\cos^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta}$

$\sin^2\theta$

B. Pythagorean Identity

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12. $\left(\cos^2\theta - \sin^2\theta\right)^2$

$\left(\cos^2\theta + \sin^2\theta\right)\left(\cos^2\theta - \sin^2\theta\right)$

1

C. Pythagorean Identity

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13. $\frac{15-4\sqrt{39}}{40}$

$\frac{-20-3\sqrt{39}}{40}$

$\frac{15+4\sqrt{39}}{40}$

$-\frac{768+125\sqrt{39}}{49}$

14.

A. $$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta} = \frac{\frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\sin\alpha\sin\beta}}{\frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\sin\beta}}$$

$$= \frac{\frac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta} - \frac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}}{\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} + \frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}} = \frac{\cot\beta - \cot\alpha}{\cot\alpha\cot\beta + 1}$$

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15. $\frac{63}{65}$

16.
B. $\dfrac{\sqrt{1-v^2}+uv}{\sqrt{1+u^2}}, \ -\infty < u < \infty, \ -1 \le v \le 1$

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17. $-\dfrac{4}{5}$

18.
A. $\tan(12\theta+6\theta) = \dfrac{\tan(12\theta)+\tan(6\theta)}{1-\tan(12\theta)\tan(6\theta)} = \dfrac{\dfrac{2\tan(6\theta)}{1-\tan^2(6\theta)}+\tan(6\theta)}{1-\dfrac{2\tan(6\theta)}{1-\tan^2(6\theta)}\tan(6\theta)} = \dfrac{3\tan(6\theta)-\tan^3(6\theta)}{1-3\tan^2(6\theta)}$

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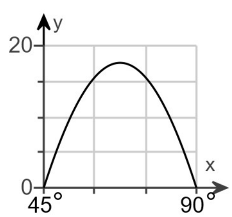
19. A. $\theta = \left\{ \dfrac{\pi}{3}, \dfrac{5\pi}{3}, \pi \right\}$ radians

(Simplify your answer. Type an exact answer, using π as needed. Type your answer in radians. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

20.
A. $R(\theta) = \dfrac{v_0{}^2\sqrt{2}}{32}[\sin(2\theta) - \cos(2\theta) - 1]$

67.5

17.59

A. 

67.5

17.59

(1) match

21.
$2\sin(4\theta)\sin\dfrac{5\theta}{2}$

22.
A. $\dfrac{\cos y + \cos x}{\cos y - \cos x} = \dfrac{2\cos\dfrac{y+x}{2}\cos\dfrac{y-x}{2}}{-2\sin\dfrac{y+x}{2}\sin\dfrac{y-x}{2}} = -\cot\dfrac{y+x}{2}\cot\dfrac{y-x}{2}$

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23. $\sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin\alpha\cos\beta - \cos\alpha\sin\beta$

$2\sin\alpha\cos\beta$

$\sin\alpha\cos\beta$
