Mini-Lecture 7.1

The Inverse Sine, Cosine, and Tangent Functions

Learning Objectives:

1. Find the Exact Value of an Inverse Sine Function (p. 441)
2. Find an Approximate Value of an Inverse Sine Function (p. 442)
3. Use Properties of Inverse functions to Find Exact Values of Certain Composite Functions (p. 443)
4. Find the Inverse Function of a Trigonometric Function (p. 449)
5. Solve Equations Involving Trigonometric Functions (p. 450)

Examples:

1. Use a calculator to find the value of $\tan^{-1}\left(-\frac{\sqrt{2}}{5}\right)$. Round to two decimal places.

2. Evaluate $\cos^{-1}\left(\cos\frac{9\pi}{4}\right)$. Give an exact value.

3. If $f(x) = 3\tan x - 2$, find the inverse function of *f*.

4. Solve the equation : $2\cos^{-1}(3x) = \pi$.

Mini Lecture 7.1

$y = \sin^{-1} x$ means $x = \sin y$

$-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$

$y = \cos^{-1} x$ means $x = \cos y$

$-1 \le x \le 1$ and $0 \le y \le \pi$

$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x$, where $-\pi/2 \le x \le \pi/2$

$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x$, where $-1 \le x \le 1$

$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x$, where $0 \le x \le \pi$

$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x$, where $-1 \le x \le 1$

mini Lecture 7.1 continued

$y = \tan^{-1} x$ means $x = \tan y$

$-\infty < x < \infty$ and $-\pi/2 < y < \pi/2$

$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x$, where $-\pi/2 < x < \pi/2$

$f(f^{-1}(x)) = \tan(\tan^{-1} x) = x$, where $-\infty < x < \infty$

1. use calculator $\tan^{-1}\left(-\frac{\sqrt{2}}{5}\right) = \boxed{-.28}$

2. $\cos^{-1}\left(\cos \frac{9\pi}{4}\right)$ $\quad * \; 0 \le x \le \pi$

$* \; \frac{9\pi}{4} - 2\pi = \frac{\pi}{4}$

$\cos^{-1}(\cos \pi/4) = \boxed{\pi/4}$

7.1 mini Lecture continued

(3) $f(x) = 3\tan x - 2$, find inverse

* flip x and y

$$x = 3\tan y - 2$$

$$\frac{x+2}{3} = \tan y$$

$$y = \tan^{-1}\left(\frac{x+2}{3}\right)$$

$$\boxed{f^{-1}(x) = \tan^{-1}\left(\frac{x+2}{3}\right)}$$

(4) $2\cos^{-1}(3x) = \pi$

$$\cos^{-1}(3x) = \pi/2$$

$$\cos(\pi/2) = 3x$$

$$0 = 3x$$

$$\boxed{x = 0}$$

Mini-Lecture 7.2

The Inverse Trigonometric Functions (Continued)

Learning Objectives:

1. Find the Exact Value of Expressions Involving the Inverse Sine, Cosine, and Tangent Functions (p. 453)
2. Define the Inverse Secant, Cosecant, and Cotangent Functions (p. 455)
3. Use a Calculator to Evaluate $\sec^{-1} x$, $\csc^{-1} x$, and $\cot^{-1} x$ (p. 455)
4. Write a Trigonometric Expression as an Algebraic Expression (p. 456)

Examples:

1. Find the exact value of $\cot\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$.

2. Evaluate $\sec\left(\tan^{-1}\left(-\frac{4}{7}\right)\right)$. Give an exact value.

3. Use a calculator to find the value of $\csc^{-1}\left(-\frac{5}{4}\right)$.

4. Write $\cos\left(\sin^{-1}\frac{2}{u}\right)$ as an algebraic expression in u.

7.2 mini lecture

$y = \sec^{-1} x$ means $x = \sec y$

$|x| \geq 1$ and $0 \leq y \leq \pi$ $\quad y \neq \pi/2$

$y = \csc^{-1} x$ means $x = \csc y$

$|x| \geq 1$ and $-\pi/2 \leq y \leq \pi/2$ $\quad y \neq 0$

$y = \cot^{-1} x$ means $x = \cot y$

$-\infty < x < \infty$ and $0 < y < \pi$

① $\cot\left(\cos^{-1}(-1/2)\right)$

$* \cos\theta = -\dfrac{1}{2} = \dfrac{x}{r}$

$\cot = \dfrac{\cos}{\sin} = \dfrac{x}{y} = \dfrac{-1}{\sqrt{3}}$

$= \boxed{\dfrac{-\sqrt{3}}{3}}$

$x^2 + y^2 = r^2$

$(-1)^2 + y^2 = (2)^2$

$y^2 = 4 - 1$

$y = \pm\sqrt{3}$

$y = \sqrt{3}$

$* \cos\theta$

$0 \leq \theta \leq \pi$

7.2 mini lecture continued

② $\sec\left(\tan^{-1}\left(-\frac{4}{7}\right)\right)$

$* \tan\theta = -\frac{4}{7} = \frac{y}{x}$

$x^2+y^2=r^2$

$(7)^2+(-4)^2=r^2$

$r^2=65$

$r=\pm\sqrt{65}$

$\sec = \frac{1}{\cos} = \frac{1}{\left(\frac{x}{r}\right)} = \frac{1}{\left(7/\sqrt{65}\right)}$

$= \boxed{\frac{\sqrt{65}}{7}}$

③ use calculator $\csc^{-1}\left(-5/4\right)$

$\csc\theta = -\frac{5}{4}$

$\frac{1}{\sin\theta} = -\frac{5}{4}$

$\sin\theta = -4/5$

$\sin^{-1}\left(-\frac{4}{5}\right) = \theta$

$\theta = \boxed{-.93}$

7.2 mini lecture continued

④ $\cos\left(\sin^{-1}\frac{2}{u}\right)$

Get cos in terms of sin

$*$ let $\sin^{-1}\frac{2}{u} = \theta$

$$\sin\theta = \frac{2}{u}$$

$$\cos(\theta) = \sqrt{\cos^2\theta} = \sqrt{1-\sin^2\theta}$$

$$= \sqrt{1-\left(\frac{2}{u}\right)^2}$$

$$= \sqrt{1-\frac{4}{u^2}}$$

$$= \sqrt{\frac{u^2-4}{u^2}}$$

$$= \boxed{\frac{\sqrt{u^2-4}}{u}}$$

Mini-Lecture 7.3
Trigonometric Equations

Learning Objectives:

1. Solve Equations Involving a Single Trigonometric Function (p. 459)
2. Solve Trigonometric Equations Using a Calculator (p. 462)
3. Solve Trigonometric Equations Quadratic in Form (p. 462)
4. Solve Trigonometric Equations Using Fundamental Identities (p. 463)
5. Solve Trigonometric Equations Using a Graphing Utility (p. 464)

Examples:

Solve each equation on the interval $0 \le \theta < 2\pi$.

1. $4\sin^2\theta - 3 = 0$

2. $2\cos^2 x + 2\cos x + \sqrt{2}\cos x = -\sqrt{2}$

3. $\cot^2 x - \csc x - 1 = 0$

4. Use a calculator to solve the equation on the interval $0 \le \theta < 2\pi$.

   $5\csc\theta = -6$

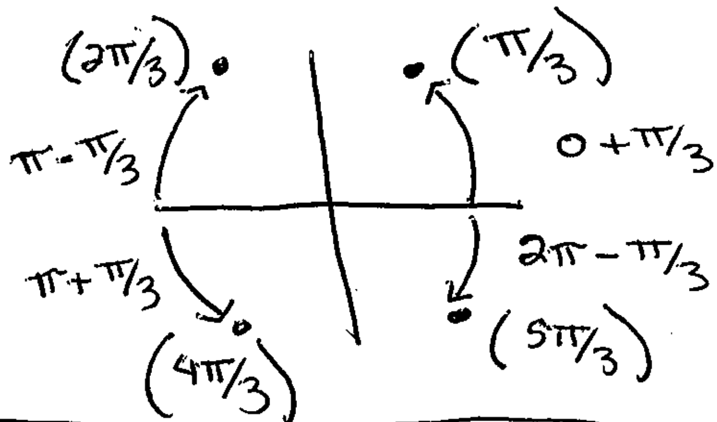
7.3 mini lecture

① $4\sin^2\theta - 3 = 0 \qquad 0 \le \theta < 2\pi$

$4\sin^2\theta = 3$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \frac{\pm\sqrt{3}}{2}$



$$\boxed{\theta = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}}$$

② $2\cos^2x + 2\cos x + \sqrt{2}\cos x = -\sqrt{2}$

$2\cos^2x + 2\cos x + \sqrt{2}\cos x + \sqrt{2} = 0$

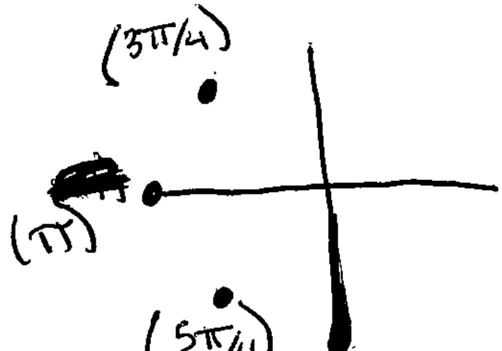
Grouping

$2\cos x(\cos x + 1) + \sqrt{2}(\cos x + 1) = 0$

$(2\cos x + \sqrt{2})(\cos x + 1) = 0$

$2\cos x + \sqrt{2} = 0 \qquad \cos x + 1 = 0$

$\cos x = \frac{-\sqrt{2}}{2} \qquad \cos x = -1$



$$\boxed{x = \left\{\frac{3\pi}{4}, \pi, \frac{5\pi}{4}\right\}}$$

7.3 mini Lecture continued

③ $\cot^2 x - \csc x - 1 = 0$

$(\csc^2 x - 1) - \csc x - 1 = 0$

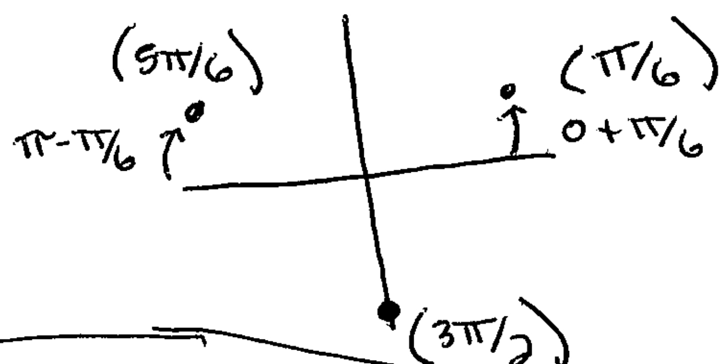
$\csc^2 x - \csc x - 2 = 0$

$(\csc x - 2)(\csc x + 1) = 0$

$\csc x = 2 \qquad \csc x = -1$

$\frac{1}{\sin x} = 2 \qquad \frac{1}{\sin x} = -1$

$\sin x = \frac{1}{2} \qquad \sin x = -1$



$$\boxed{x = \left\{ \pi/6,\ 5\pi/6,\ 3\pi/2 \right\}}$$

7.3 mini lecture continued

(4) use calculator $\qquad 0 \le \theta < 2\pi$

$$5\csc\theta = -6$$

$$\csc\theta = \frac{-6}{5}$$

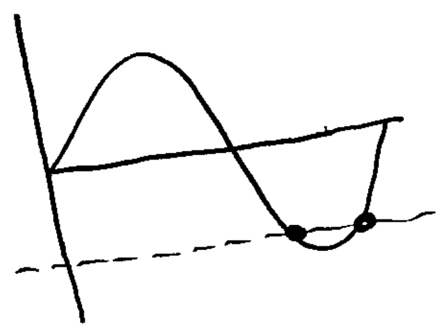
$$\frac{1}{\sin\theta} = -\frac{6}{5}$$

$$\sin\theta = \frac{-5}{6}$$

$Y_1 = \sin x$

$Y_2 = -5/6$

Intersect



$$\boxed{x = \{4.13,\ 5.30\}}$$

Mini-Lecture 7.4
Trigonometric Identities

Learning Objectives:

1. Use Algebra to Simplify Trigonometric Expressions (p. 470)
2. Establish Identities (p. 470)

Examples:

1. Rewrite $\cos^2 x + \cot^2 x + \sin^2 x$ in terms of $\csc x$.

2. Simplify by rewriting $\frac{\sin x}{\cos x + 1} + \frac{\cos x - 1}{\sin x}$ over a common denominator.

Verify the following identities.

3. $\frac{1}{\sec x + \tan x} + \frac{1}{\sec x - \tan x} = 2\sec x$

4. $\sin x = -\cos x \tan(-x)$

7.4 mini lecture

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\csc\theta = \dfrac{1}{\sin\theta}$ $\qquad$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\qquad$ $\cot\theta = \dfrac{1}{\tan\theta}$

$$\left.\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \tan^2\theta + 1 &= \sec^2\theta \\ \cot^2\theta + 1 &= \csc^2\theta \end{aligned}\right] \text{pythagorean Identities}$$

$\sin(-\theta) = -\sin\theta$

$\csc(-\theta) = -\csc\theta$

$\cos(-\theta) = \cos\theta$

$\sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

7.4 mini lecture continued

① Rewrite $\cos^2 x + \cot^2 x + \sin^2 x$ in terms of $\csc x$

$$(1-\sin^2 x) + (\csc^2 x - 1) + \left(\frac{1}{\csc^2 x}\right)$$

$$\left(1 - \frac{1}{\csc^2 x}\right) + (\csc^2 x - 1) + \left(\frac{1}{\csc^2 x}\right)$$

*cancel out

$$\boxed{\csc^2 x}$$

② $\frac{\sin x}{\cos x + 1} + \frac{\cos x - 1}{\sin x}$

$$\frac{\sin^2 x + (\cos x + 1)(\cos x - 1)}{(\cos x + 1)(\sin x)}$$

$$\frac{\sin^2 x + \cos^2 x - 1}{(\cos x + 1)(\sin x)} = \frac{(1) - 1}{(\cos x + 1)(\sin x)}$$

$$= \boxed{0}$$

7.4 mini lecture continued

(3) $$\frac{1}{\sec x+\tan x}+\frac{1}{\sec x-\tan x}=2\sec x$$

$$\frac{(\sec x-\tan x)+(\sec x+\tan x)}{(\sec x+\tan x)(\sec x-\tan x)}=2\sec x$$

$$\frac{2\sec x}{\sec^2 x-\tan^2 x}=2\sec x$$

$$\frac{2\sec x}{\sec^2 x-(\sec^2 x-1)}=2\sec x$$

$$\frac{2\sec x}{1}=2\sec x$$

$$\boxed{2\sec x=2\sec x}$$ ✓

7.4 mini lecture continued

(4) $\sin x = -\cos x \tan(-x)$

$* \tan(-x) = -\tan x$

$\sin x = \cos x \tan x$

$\sin x = \cos x \left(\frac{\sin x}{\cos x}\right)$

$\boxed{\sin x = \sin x}$ ✓

Mini-Lecture 7.5
Sum and Difference Formulas

Learning Objectives:

1. Use Sum and Difference Formulas to Find Exact Values (p. 477)
2. Use Sum and Difference Formulas to Establish Identities (p. 478)
3. Use Sum and Difference Formulas Involving Inverse Trigonometric Functions (p. 482)
4. Solve Trigonometric Equations Linear in Sine and Cosine (p. 483)

Examples:

1. Write the expression as the sine, cosine, or tangent of an angle. Then find the exact value of the expression.

$$\cos\frac{7\pi}{12}\sin\frac{\pi}{12}+\sin\frac{7\pi}{12}\cos\frac{\pi}{12}$$

2. Find the exact value of the following under the given conditions:

$$\cos(\alpha+\beta) \qquad \sin(\alpha+\beta) \qquad \tan(\alpha+\beta)$$

$\tan\alpha=\frac{3}{4}$, α lies in quadrant III, and $\sin\beta=\frac{8}{17}$, β lies in quadrant II.

3. Find the exact value of $\tan\left(\cos^{-1}\frac{2}{3}+\sin^{-1}\left(-\frac{1}{5}\right)\right)$.

4. Establish the identity: $\dfrac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\dfrac{1-\tan\alpha\tan\beta}{1+\tan\alpha\tan\beta}$.

5. Solve the equation on the interval $0\le x<2\pi$: $\sin\theta+\sqrt{3}\cos\theta=1$

7.5 mini Lecture

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\sin\left(\pi/2 - \theta\right) = \cos\theta$$

7.5 mini lecture continued

① $\cos\frac{7\pi}{12}\sin\frac{\pi}{12} + \sin\frac{7\pi}{12}\cos\frac{\pi}{12}$

rewrite to match formula

$\sin\frac{7\pi}{12}\cos\frac{\pi}{12} + \cos\frac{7\pi}{12}\sin\frac{\pi}{12}$

$A = \frac{7\pi}{12}$

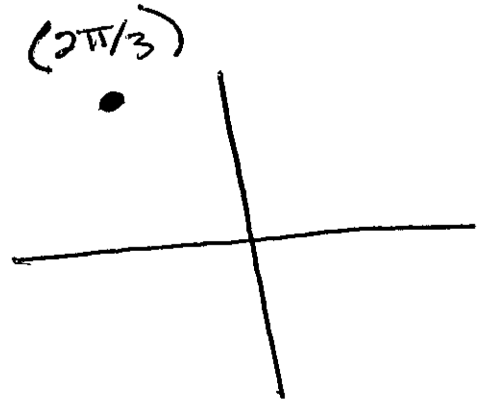
$B = \frac{\pi}{12}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin\left(\frac{7\pi}{12} + \frac{\pi}{12}\right)$

$\sin\left(\frac{8\pi}{12}\right)$

$\sin\frac{2\pi}{3} = \boxed{\frac{\sqrt{3}}{2}}$

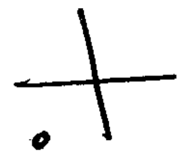


7.5 mini lecture continued

(2) $\tan A = \frac{3}{4}$ in Q3

$\sin B = \frac{8}{17}$ in Q2

A



$x = -4$

$y = -3$

$x^2 + y^2 = r^2$

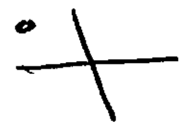
$r = \sqrt{5}$

$\sin A = \frac{-3}{5}$

$\cos A = -\frac{4}{5}$

$\tan A = \frac{3}{4}$

B



$y = 8$

$r = 17$

$x^2 + y^2 = r^2$

$x^2 + 8^2 = 17^2$

$x = -15$

$\sin B = \frac{8}{17}$

$\cos B = -\frac{15}{17}$

$\tan B = -\frac{8}{15}$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\left(-\frac{4}{5}\right)\left(-\frac{15}{17}\right) - \left(-\frac{3}{5}\right)\left(\frac{8}{17}\right) = \boxed{\frac{84}{85}}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\left(-\frac{3}{5}\right)\left(\frac{-15}{17}\right) + \left(-\frac{4}{5}\right)\left(\frac{8}{17}\right) = \boxed{\frac{13}{85}}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{(3/4) + (-8/15)}{1 - (3/4)(-8/15)}$$

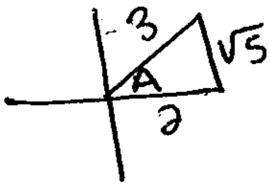
$$\boxed{13/84}$$

7.5 mini lecture continued

(3) exact value $\tan\left(\cos^{-1}(2/3) + \sin^{-1}(-1/5)\right)$

* Get $\cos^{-1}$ and $\sin^{-1}$ in terms of tan

$\cos\theta = \frac{2}{3} = \frac{x}{r}$



$x^2 + y^2 = r^2$

$2^2 + y^2 = 3^2$

$0 \le \theta \le \pi$

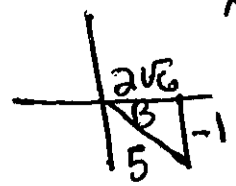
Q1

$y = \sqrt{5}$

$\tan A = \frac{\sqrt{5}}{2}$

$\sin\theta = -\frac{1}{5} = \frac{y}{r}$

$x^2 + y^2 = r^2$



$x^2 + (-1)^2 = 5^2$

$x = \sqrt{24}$

$x = 2\sqrt{6}$

$-\frac{\pi}{2} \le \theta \le \pi/2$

Q4

$\tan B = -\frac{1}{2\sqrt{6}}$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{\sqrt{5}}{2} + \left(-\frac{1}{2\sqrt{6}}\right)}{1 - \left(\frac{\sqrt{5}}{2}\right)\left(-\frac{1}{2\sqrt{6}}\right)}$$

$$= \boxed{\frac{12\sqrt{5} - 2\sqrt{6}}{24 + \sqrt{30}}}$$

7.5 mini lecture continued

④ $$\frac{\cos(A+B)}{\cos(A-B)} = \frac{1-\tan A\tan B}{1+\tan A\tan B}$$

$$\frac{\cos A\cos B - \sin A\sin B}{\cos A\cos B + \sin A\sin B} =$$

*divide numerator & denominator by $\cos A\cos B$

$$\frac{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} + \frac{\sin A\sin B}{\cos A\cos B}} =$$

$$\frac{1 - \frac{\sin A}{\cos A}\cdot\frac{\sin B}{\cos B}}{1 + \frac{\sin A}{\cos A}\cdot\frac{\sin B}{\cos B}} =$$

$$\boxed{\frac{1-\tan A\tan B}{1+\tan A\tan B}}$$ ✓

7.5 mini lecture continued

⑤ $0 \le x < 2\pi$ solve $\sin\theta + \sqrt{3}\cos\theta = 1$

$\sqrt{3}\cos\theta = 1 - \sin\theta$

*square both sides

$3\cos^2\theta = 1 - 2\sin\theta + \sin^2\theta$

$0 = (\sin^2\theta + \cos^2\theta) - 2\sin\theta + \sin^2\theta - 3\cos^2\theta$

$0 = 2\sin^2\theta - 2\sin\theta - 2\cos^2\theta$

*divide by 2

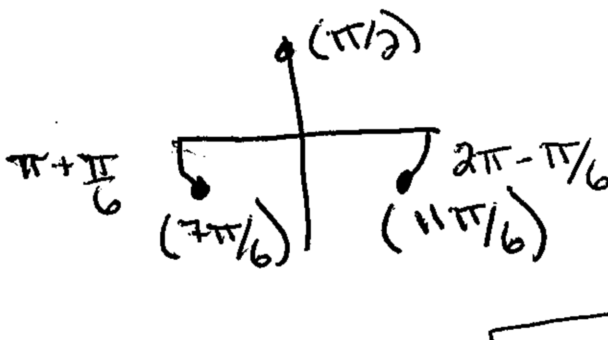
$0 = \sin^2\theta - \sin\theta - \cos^2\theta$

$0 = \sin^2\theta - \sin\theta - (1 - \sin^2\theta)$

$0 = 2\sin^2\theta - \sin\theta - 1$

$0 = (2\sin\theta + 1)(\sin\theta - 1)$

$\sin\theta = -1/2$ $\quad\quad$ $\sin\theta = 1$



$\frac{7\pi}{6}$ does not work

$$\boxed{\{\pi/2,\ 11\pi/6\}}$$

Mini-Lecture 7.6

Double-angle and Half-angle Formulas

Learning Objectives:

1. Use Double-angle Formulas to Find Exact Values (p. 488)
2. Use Double-angle Formulas to Establish Identities (p. 489)
3. Use Half-angle Formulas to Find Exact Values (p. 492)

Examples:

1. If $\tan\theta = 4$ and θ lies in quadrant III, find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

2. Use a half-angle formula to find the exact value of $\tan\frac{5\pi}{12}$.

3. If $\csc\alpha = -\frac{25}{24}$ and α is in quadrant IV, find $\sin\frac{\alpha}{2}$, $\cos\frac{\alpha}{2}$, and $\tan\frac{\alpha}{2}$.

4. Find the exact value of $\sin\left(2\tan^{-1}\left(-\frac{3}{4}\right)\right)$.

7.6 mini lecture

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$\cos(2\theta) = 1 - 2\sin^2\theta$$
$$\cos(2\theta) = 2\cos^2\theta - 1$$
$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Double Angle

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$
$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$
$$\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

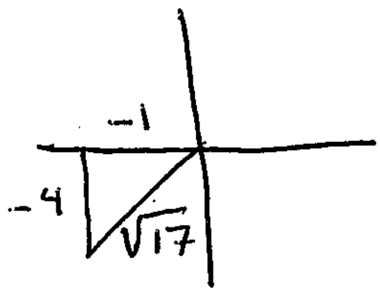
Half Angle

all 3

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} =$$
$$= \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

7.6 mini lecture continued

(1) $\tan\theta = 4$ $\quad\theta$ in Q3



$x^2+y^2=r^2$

$(-1)^2+(-4)^2=r^2$

$r=\sqrt{17}$

$\sin\theta = \frac{-4}{\sqrt{17}}$

$\cos\theta = -\frac{1}{\sqrt{17}}$

$\tan\theta = 4$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(-\frac{4}{\sqrt{17}}\right)\left(-\frac{1}{\sqrt{17}}\right) = \boxed{\frac{8}{17}}$$

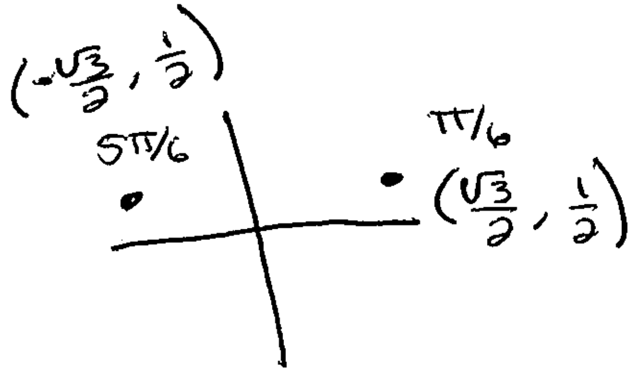
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \left(-\frac{1}{\sqrt{17}}\right)^2 - \left(-\frac{4}{\sqrt{17}}\right)^2 = \boxed{-\frac{15}{17}}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$
$$= \frac{2(4)}{1-(4)^2} = \boxed{-\frac{8}{15}}$$

7.6 mini lecture continued

② $\tan \frac{5\pi}{12}$ * use $\frac{1}{2}$ angle

$$\tan\left(\frac{5\pi/6}{2}\right)$$



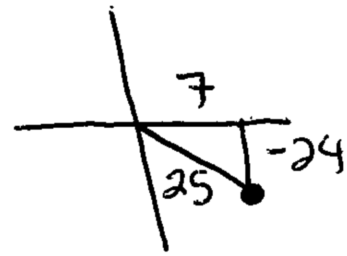
$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$

$$= \frac{1-\cos(5\pi/6)}{\sin(5\pi/6)}$$

$$= \frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{1/2} = \boxed{2+\sqrt{3}}$$

7.6 mini lecture continued

(3) $\csc\theta = -\dfrac{25}{24}$ $\quad \theta$ in Q4



$$\frac{1}{\sin\theta} = \frac{-25}{24}$$

$$\sin\theta = -\frac{24}{25}$$

$$x^2 + y^2 = r^2$$

$$x^2 + (-24)^2 = (25)^2$$

$$\cos\theta = \frac{7}{25}$$

$$x = 7$$

$$y = -24$$

$$r = 25$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} = \pm\sqrt{\frac{1-7/25}{2}} = \boxed{\frac{3}{5}}$$

Q2

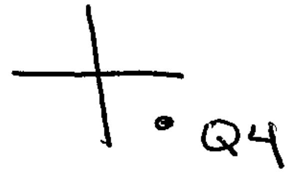
$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} = \pm\sqrt{\frac{1+7/25}{2}} = \boxed{-\frac{4}{5}} \quad \text{Q2}$$

$$\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{3/5}{-4/5} = \boxed{-3/4} \quad \text{Q2}$$

7.6 mini lecture continued

④ find exact value of

$\sin\left(2\tan^{-1}\left(-\frac{3}{4}\right)\right)$



$* \tan\theta = -\frac{3}{4}$

$x^2+y^2=r^2$

$(4)^2+(-3)^2=r^2$

$r=5$

$* \sin\theta = \frac{-3}{5}$

$* \cos\theta = \frac{4}{5}$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$= 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = \boxed{-\frac{24}{25}}$$

Mini-Lecture 7.7

Product-to-Sum and Sum-to-Product Formulas

Learning Objectives:

1. Express Products as Sums (p. 498)
2. Express Sums as Products (p. 499)

Examples:

1. Use the appropriate formula to express $\cos\frac{5\theta}{4}\sin\frac{\theta}{4}$ as a sum or difference.

2. Use the appropriate formula to express $\sin 10\theta \sin 3\theta$ as a sum or difference.

3. Express $\cos 8\theta - \cos 2\theta$ as a product.

4. Establish the identity: $\dfrac{\sin 3\theta - \sin\theta}{\cos 3\theta - \cos\theta} = -\cot 2\theta$

7.7 mini lecture

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\sin\frac{A-B}{2}\cos\frac{A+B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

7.7 mini lecture continued

(1) $\cos\left(\frac{5\theta}{4}\right)\sin\left(\frac{\theta}{4}\right)$

$$\cos A \sin B = \frac{1}{2}\sin(A+B) - \frac{1}{2}\sin(A-B)$$

$$= \frac{1}{2}\sin\left(\frac{5\theta}{4}+\frac{\theta}{4}\right) - \frac{1}{2}\sin\left(\frac{5\theta}{4}-\frac{\theta}{4}\right)$$

$$= \boxed{\frac{1}{2}\left(\sin\frac{3\theta}{2} - \sin\theta\right)}$$

(2) $\sin 10\theta \sin 3\theta$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$= \frac{1}{2}\left[\cos(10\theta-3\theta) - \cos(10\theta+3\theta)\right]$$

$$= \boxed{\frac{1}{2}\left[\cos 7\theta - \cos 13\theta\right]}$$

7.7 mini lecture continued

③ $\cos 8\theta - \cos 2\theta$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$= -2\sin\frac{8\theta+2\theta}{2}\sin\frac{8\theta-2\theta}{2}$$

$$= \boxed{-2\sin 5\theta \sin 3\theta}$$

④ $$\frac{\sin 3\theta - \sin\theta}{\cos 3\theta - \cos\theta} = -\cot 2\theta$$

$$\frac{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}{-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}$$

$$\frac{2\sin\theta\cos 2\theta}{-2\sin 2\theta\sin\theta}$$

$$-\frac{\cos 2\theta}{\sin 2\theta}$$

$$\boxed{-\cot 2\theta} \checkmark$$