Mini-Lecture 6.1
Angles and Their Measure

Learning Objectives:

1. Convert between Decimals and Degrees, Minutes, Seconds Measures for Angles (p. 357)
2. Find the Arc Length of a Circle (p. 359)
3. Convert from Degrees to Radians and from Radian to Degrees (p. 359)
4. Find the Area of a Sector of a Circle (p. 362)
5. Find the Linear Speed of an Object Traveling in Circular Motion (p. 363)

Examples:

1. Convert $87^\circ 32' 15''$ to a decimal in degrees.

2. Convert $-\frac{11\pi}{3}$ to degree measure.

3. If $\theta = \frac{1}{3}$ radians $s = 12$ meters and s denotes the length of the arc of a circle of radius r subtended by the central angle θ, find r.

4. A water wheel has a radius of 15 feet. The wheel is rotating at 9 revolutions per minute. Find the linear speed of the water.

6.1 mini lecture

1 counterclockwise revolution $= 360°$

$1° = 60'$

$1' = 60''$

$$\boxed{\frac{\text{Degree}}{180°} = \frac{\text{Radian}}{\pi}}$$

① Convert $87° 32' 15''$ to a decimal in degrees

$$87 + \frac{32}{60} + \frac{15}{60(60)} = \boxed{87.5375°}$$

② Convert $\frac{-11\pi}{3}$ to degree measures

$$\frac{\text{degree}}{180°} = \frac{-11\pi/3}{\pi} \qquad \boxed{-660°}$$

6.1 mini lecture continued

$S = r\theta$ Arc Length

1 rev = 2π radians

$180^\circ = \pi$ radians

$1^\circ = \pi/180$ radian

1 rad = $\frac{180}{\pi}$ degrees

3 $\theta = 1/3$ radians
$S = 12$ meters

$S = r\theta$
$12 = r(1/3)$
$\boxed{r = 36 \text{ meters}}$

6.1 mini lecture continued

$A = \frac{1}{2}r^2\theta$ Area of sector

$V = \frac{s}{t}$ Linear speed

$\omega = \frac{\theta}{t}$ angular speed (ω is omega)

$V = r\omega$ Linear speed

(4) radius 15 ft, rotating 9 rev/min

Find linear speed.

$$V = r\omega$$

$$V = r\left(\frac{\theta}{t}\right)$$

$$V = 15\left(\frac{9(2\pi)}{1\,\text{min}}\right) = \boxed{270\pi \text{ ft/min}}$$

Mini-Lecture 6.2

Trigonometric Functions: Unit Circle Approach

Learning Objectives:

1. Find the Exact Values of the Trigonometric Functions Using a Point on the Unit Circle (p. 370)
2. Find the Exact Values of the Trigonometric Functions of Quadrantal Angles (p. 371)
3. Find the Exact Values of the Trigonometric Functions of $\frac{\pi}{4} = 45^\circ$ (p. 373)
4. Find the Exact Values of the Trigonometric Functions of $\frac{\pi}{6} = 30^\circ$ and $\frac{\pi}{3} = 60^\circ$ (p. 374)
5. Find the Exact values of the Trigonometric Functions for Integer Multiples of $\frac{\pi}{6} = 30^\circ$, $\frac{\pi}{4} = 45^\circ$, and $\frac{\pi}{3} = 60^\circ$ (p. 374)
6. Use a Calculator to Approximate the Values of the Trigonometric Functions of Acute Angles (p. 378)
7. Use a Circle of Radius r to Evaluate the Trigonometric Functions (p. 379)

Examples:

1. Use a calculator to find the approximate value of $\cot\frac{\pi}{9}$.

2. If $f(\theta) = 2\sin\theta - \sin 2\theta$, find $f\left(\frac{\pi}{6}\right)$. Express the answer as a single fraction.

   Do not use a calculator.

3. $P\left(-\frac{4}{5}, \frac{3}{5}\right)$ is a point on the unit circle corresponding to a real number t. Find the values of the trigonometric functions at t.

4. Find the exact value of $\tan(-675^\circ)$.

6.2 mini lecture

$\sin\theta = \frac{y}{r}$ $\qquad$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\qquad$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\qquad$ $\cot\theta = \frac{x}{y}$

$$x^2 + y^2 = r^2$$

1. $\cot\left(\pi/9\right)$ on calculator

$$\frac{1}{\tan\left(\pi/9\right)} = \boxed{2.747}$$

2. $f(\theta) = 2\sin\theta - \sin 2\theta$, find $f\left(\pi/6\right)$

$$2\sin\left(\pi/6\right) - \sin\left(2\left(\pi/6\right)\right)$$

$$2\left(1/2\right) - \left(\frac{\sqrt{3}}{2}\right) = 1 - \frac{\sqrt{3}}{2}$$

$$\boxed{\frac{2-\sqrt{3}}{2}}$$

6.2 mini lecture continued

③ $P\left(-\frac{4}{5}, \frac{3}{5}\right)$

$\sin t = \boxed{\frac{3}{5}}$

$\cos t = \boxed{-\frac{4}{5}}$

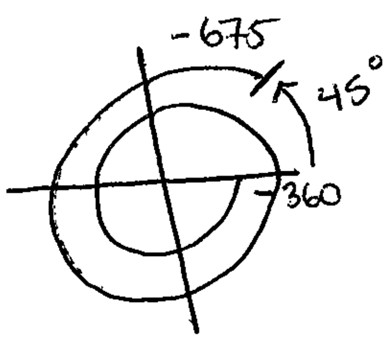
$\tan t = \frac{3/5}{-4/5} = \boxed{-\frac{3}{4}}$

$\csc t = \boxed{\frac{5}{3}} \longrightarrow \frac{1}{\sin t}$

$\sec t = \boxed{-5/4} \longrightarrow \frac{1}{\cos t}$

$\cot t = \boxed{-\frac{4}{3}} \longrightarrow \frac{1}{\tan t}$

④ Find exact value of $\tan(-675^\circ)$



$\tan(45^\circ) = \frac{\sin 45^\circ}{\cos 45^\circ}$

$= \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \boxed{1}$

Mini-Lecture 6.3

Properties of the Trigonometric Functions

Learning Objectives:

1. Know the Domain and Range of the Trigonometric Functions (p. 385)
2. Determine the Period of the Trigonometric Functions (p. 386)
3. Determine the Signs of the Trigonometric Functions in a Given Quadrant (p. 388)
4. Find the Values of the Trigonometric Functions Using Fundamental Identities (p. 389)
5. Find the Exact Values of the Trigonometric Functions of an Angle Given One of the Functions and the Quadrant of the Angle (p. 391)
6. Use Even-Odd Properties to Find the Exact Values of the Trigonometric Functions (p. 394)

Examples:

1. Use the fact that the trigonometric functions are periodic to find the exact value of $\tan(-660^\circ)$.

2. Use even and odd properties of trigonometric functions to find the value of $\cos\left(-\frac{11\pi}{6}\right)$.

3. If $\tan\theta = -\frac{5}{12}$ and $\sin\theta < 0$, find the remaining trigonometric functions.

4. Use properties of the trigonometric functions to find the exact value of $\cot 50^\circ + \cot 130^\circ$.

6.3 mini lecture

Unit Circle, radius 1

$\sin\theta = y \qquad \csc\theta = \frac{1}{y}$

$\cos\theta = x \qquad \sec\theta = \frac{1}{x}$

$\tan\theta = y/x \qquad \cot\theta = \frac{x}{y}$

Domain and Range

Sine: D (All Real)
R $(-1 \leq y \leq 1)$

Cos: D (All Real)
R $(-1 \leq y \leq 1)$

tan: D (All real, except odd integer multiples of $\pi/2$ $(90°)$)
R (All real)

Sec: D (All real, except odd integer multiples of $\pi/2$ $(90°)$)
R (All real, greater than or equal to 1 or less than or equal to -1)

csc: D (All real, except integer multiples of π $(180°)$)
R (All real, greater than or equal to 1 or less than or equal to -1)

cot: D (All real, except integer multiples of π $(180°)$)
R (All real)

6.3 mini lecture continued

$\sin(\theta + 2\pi K) = \sin\theta$

$\cos(\theta + 2\pi K) = \cos\theta$

K = any integer

period for sin, cos, sec, cos is 2π

period for tan, cot is π

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Reciprocal Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Quotient Identities

6.3 mini lecture continued

$$\left.\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \tan^2\theta + 1 &= \sec^2\theta \\ \cot^2\theta + 1 &= \csc^2\theta \end{aligned}\right\} \text{Fundamental Identities}$$

Even-odd Properties

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$

$\sec(-\theta) = \sec\theta$

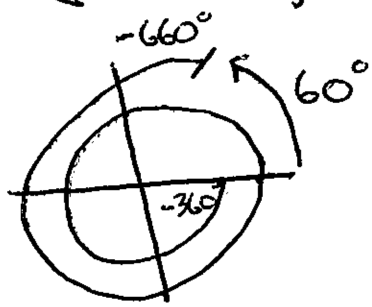
$\cot(-\theta) = -\cot\theta$

* $f(-\theta) = f(\theta)$
Even

* $f(-\theta) = -f(\theta)$
odd

6.3 mini lecture continued

① $\tan(-660°)$

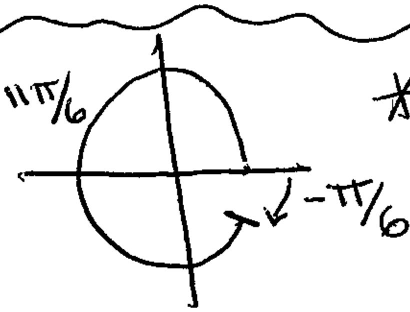


$\tan 60° = \dfrac{\sin 60°}{\cos 60°}$

$= \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \boxed{\sqrt{3}}$

② $\cos(-11\pi/6)$

$* \cos(-11\pi/6) = \cos(11\pi/6)$



$* \cos(-\pi/6) = \cos \pi/6$

$= \boxed{\dfrac{\sqrt{3}}{2}}$

6.3 mini lecture continued

(3) $\tan\theta = \frac{-5}{12}$ $\quad \sin\theta < 0$ (negative)

$$x^2 + y^2 = r^2$$

$$(12)^2 + (-5)^2 = r^2$$

$$r = 13$$

$x = 12$

$y = -5$

$r = 13$

$\sin\theta = \frac{-5}{13}$	$\frac{y}{r}$
$\cos\theta = \frac{12}{13}$	$\frac{x}{r}$
$\tan\theta = \frac{-5}{12}$	$\frac{y}{x}$
$\csc\theta = \frac{13}{-5}$	r/y
$\sec\theta = \frac{13}{12}$	r/x
$\cot\theta = \frac{-12}{5}$	$\frac{x}{y}$

(4) $\cot(50^\circ) + \cot(130^\circ)$

$*$ period $= \pi = 180^\circ$

$$\cot(50^\circ) + (\cot(130^\circ - 180^\circ))$$

$$\cot 50^\circ + \cot(-50^\circ)$$

$$\cot 50^\circ - \cot 50^\circ = \boxed{0}$$

Mini-Lecture 6.4

Graphs of the Sine and Cosine Functions

Learning Objectives:

1. Graph Functions of the Form $y = A\sin(\omega x)$ Using Transformations (p. 399)
2. Graph Functions of the Form $y = A\cos(\omega x)$ Using Transformations (p. 401)
3. Determine the Amplitude and Period of Sinusoidal Functions (p. 402)
4. Graph Sinusoidal Functions Using Key points (p. 404)
5. Find an Equation for a Sinusoidal Graph (p. 407)

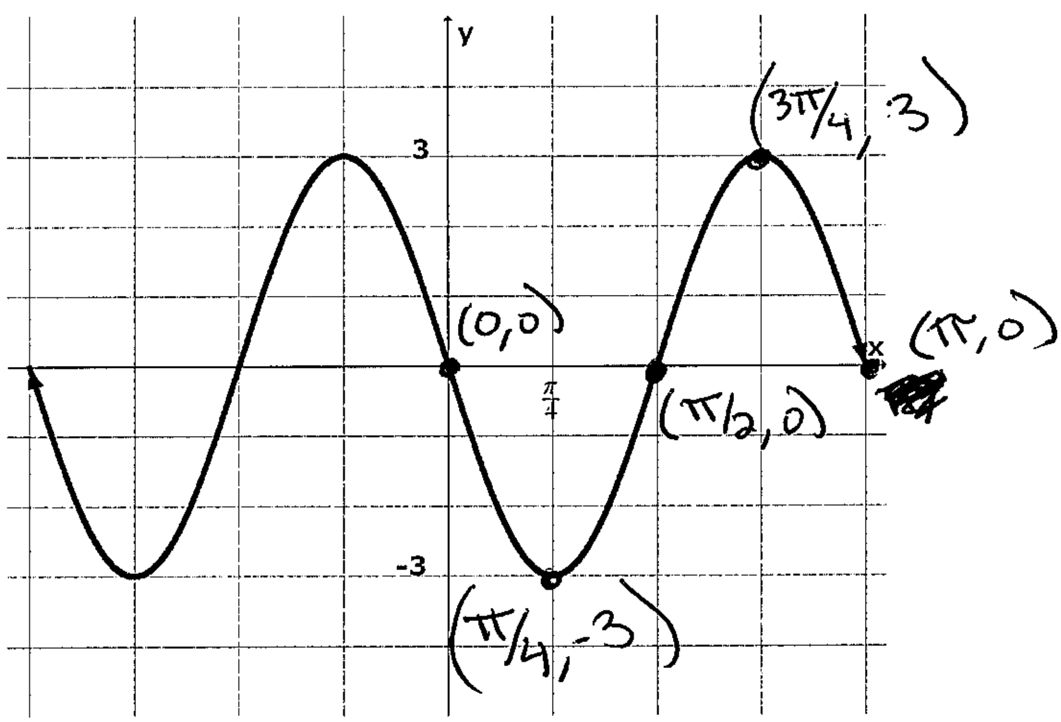
Examples:

1. Determine the amplitude and period of $y = -2\cos(\pi x)$.

Graph the following.

2. $y = 4\sin(3x)$ 3. $y = -2\cos\frac{x}{4} - 2$

4. Find the equation for the following graph.



6.4 mini lecture

$$\left.\begin{array}{l} y = A\sin(\omega x) \\ y = A\cos(\omega x) \end{array}\right] \text{ Sinusoidal Graphs}$$

Amplitude $= |A|$

Period $= T = \frac{2\pi}{\omega}$

1. $y = -2\cos(\pi x)$

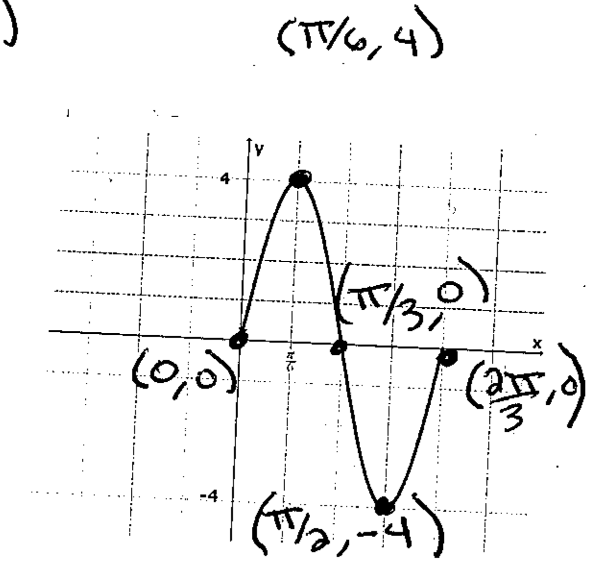
   Amplitude $= |-2| = \boxed{2}$

   period $= \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = \boxed{2}$

2. Graph $y = 4\sin(3x)$

   Amplitude $= |4| = \boxed{4}$

   period $= \frac{2\pi}{\omega} = \boxed{\frac{2\pi}{3}}$

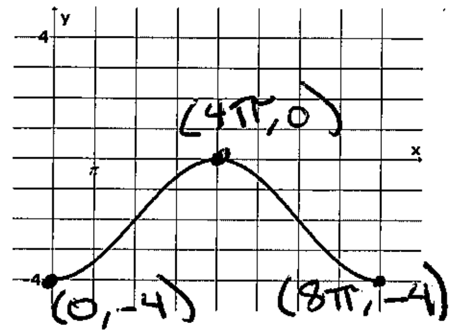


6.4 mini lecture continued

(3) $y = -2\cos\left(\frac{x}{4}\right) - 2$

Amplitude $|-2| = \boxed{2}$

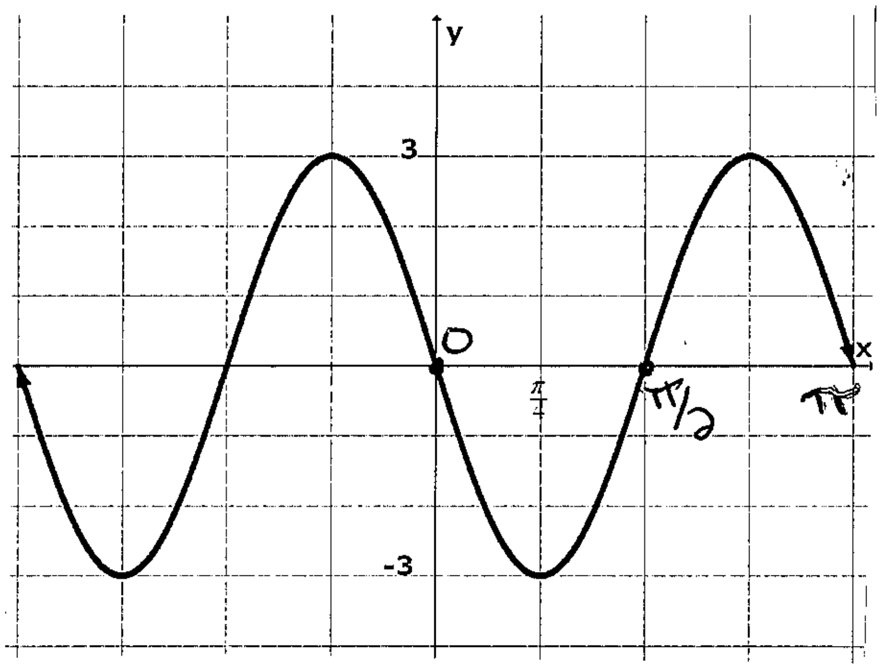
period $= \frac{2\pi}{\omega} = \frac{2\pi}{1/4} = \boxed{8\pi}$



Shift graph down 2
units due to the
-2 at end of equation

(4) find equation

$\pi = \frac{2\pi}{\omega}$ $\boxed{\omega = 2}$



Amplitude
$= \boxed{3}$

Sin starts
down so $(-)$

$y = A\sin(\omega x)$

$y = -3\sin(\pi x)$

$\boxed{y = -3\sin(2x)}$

Mini-Lecture 6.5

Graphs of the Tangent, Cotangent, Cosecant, and Secant Functions

Learning Objectives:

1. Graph Functions of the Form $y = A\tan(\omega x) + B$ and $y = A\cot(\omega x) + B$ (p. 415)
2. Graph Functions of the Form $y = A\csc(\omega x) + B$ and $y = A\sec(\omega x) + B$ (p. 417)

Examples:

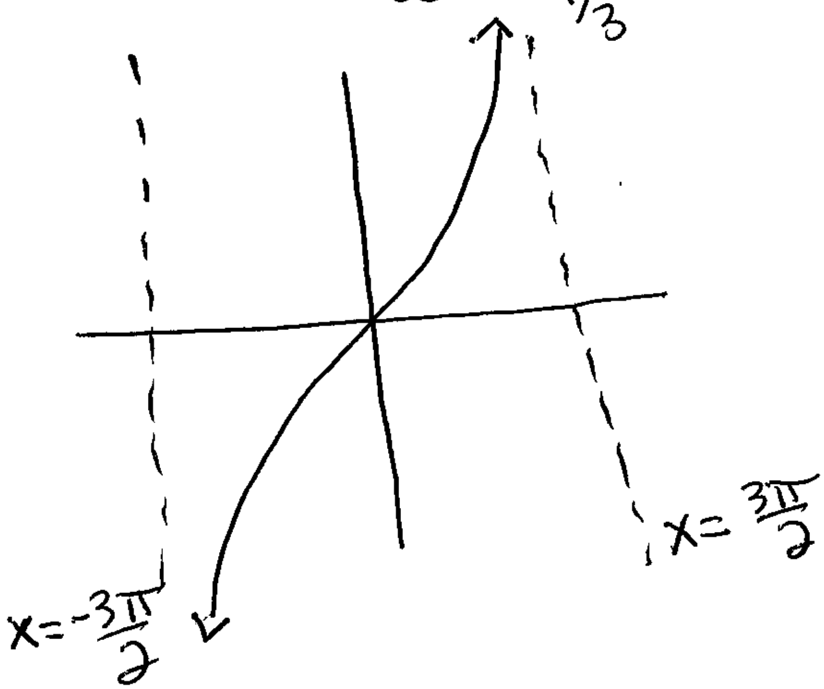
1. $y = 2\tan\left(\frac{x}{3}\right)$, $-\frac{3\pi}{2} < x < \frac{3\pi}{2}$
2. $y = \cot(\pi x) + 1$, $-2 < x < 2$
3. $y = \sec(2x)$, $-\pi \le x \le \pi$
4. $y = \csc\left(\frac{1}{2}x\right)$, $-2\pi < x < 2\pi$

6.5 mini lecture

① $y = 2\tan\left(\frac{x}{3}\right) \quad -\frac{3\pi}{2} < x < \frac{3\pi}{2}$

$\text{Amplitude} = |2| = 2$

$\text{period} = \frac{\pi}{\omega} = \frac{\pi}{1/3} = 3\pi$



* tan undefined at odd integer multiples of $\pi/2$

so $\frac{\pi/2}{1/3} = \underline{\underline{3\pi/2}}$

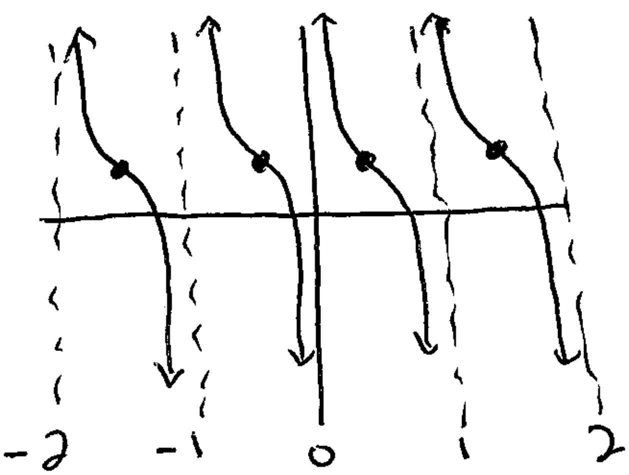
asymptote

6.5 mini lecture continued

② $y = \cot(\pi x) + 1 \qquad -2 < x < 2$

Amplitude $= |1| = 1$

period $= \dfrac{\pi}{\omega} = \dfrac{\pi}{\pi} = 1$



* cot undefined at integer multiples of π,

so $\dfrac{\pi}{\pi} = 1$ asymptote

6.5 mini lecture continued

③ $y = \sec(2x)$ $\quad -\pi \le x \le \pi$

Amplitude $= |1| = 1$

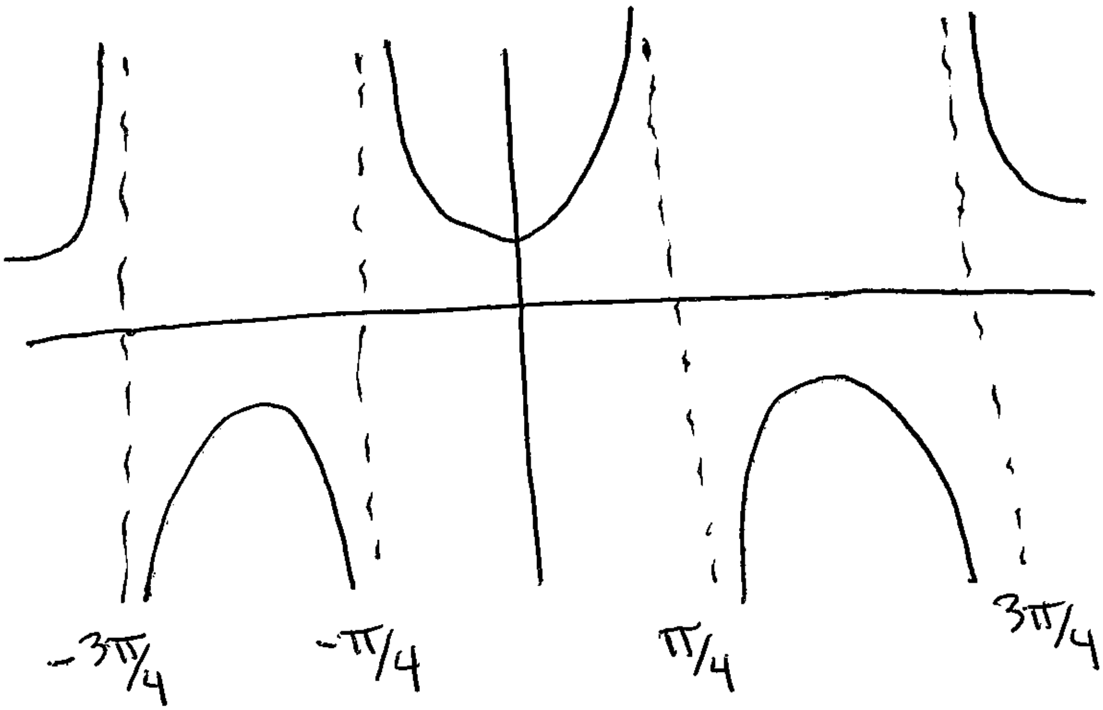
period $= \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

* sec is undefined at odd multiples of $\pi/2$,

so

$\frac{\pi/2}{2} = \underline{\underline{\pi/4}}$

asymptote



6.5 mini lecture continued

④ $y = \csc\left(\frac{1}{2}x\right)$ $-2\pi < x < 2\pi$

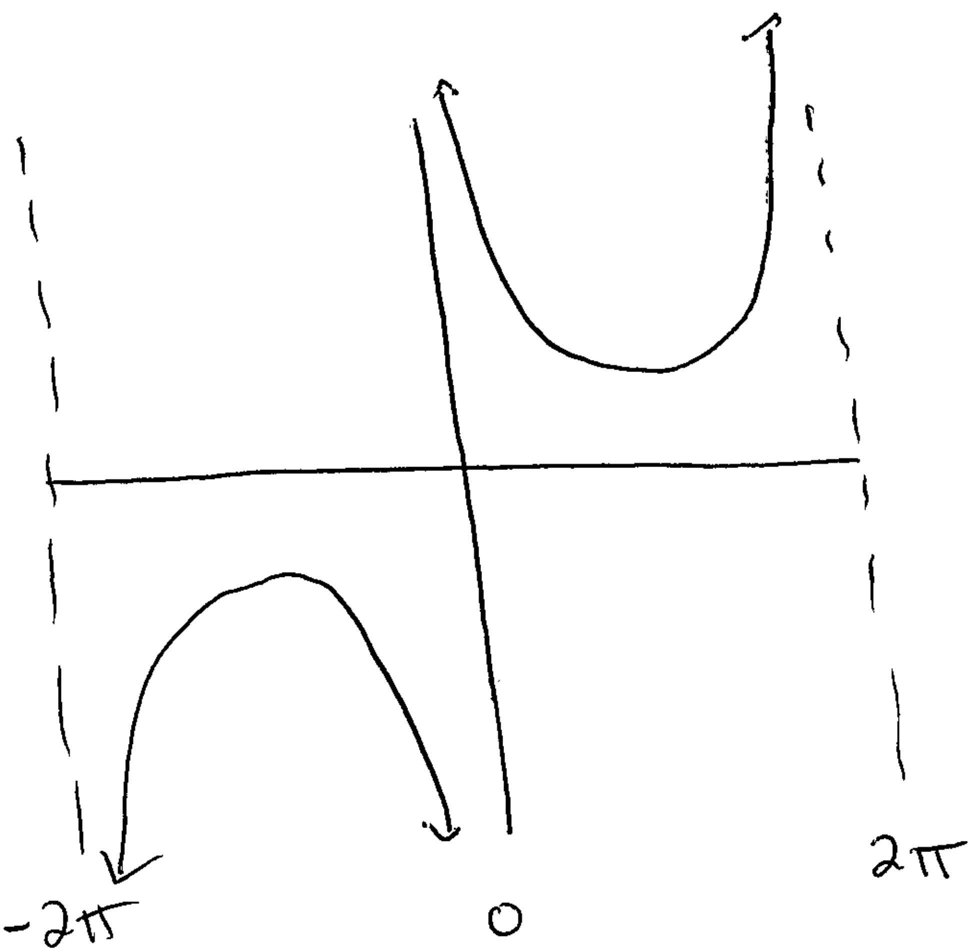
Amplitude $|A| = |1| = 1$

period $= \frac{2\pi}{\omega} = \frac{2\pi}{1/2} = 4\pi$

* csc is undefined at multiples of π, so

$\frac{\pi}{1/2} = \underline{\underline{2\pi}}$

asymptotes



Mini-Lecture 6.6

Phase Shift; Sinusoidal Curve Fitting

Learning Objectives:

1. Graph Sinusoidal Functions of the Form $y = A\sin(\omega x - \phi) + B$ (p. 420)
2. Build Sinusoidal Models from Data (p. 424)

Examples:

1. Find the amplitude, period and phase shift of $y = -2\cos\left(\frac{x}{2} - \pi\right)$.

2. Graph at least two periods of the function.

$$y = 4\sin\left(3x + \frac{\pi}{2}\right)$$

3. Write the equation of the sine function that has amplitude: $\frac{1}{2}$, period: $\frac{\pi}{4}$, and

   phase shift: $-\frac{\pi}{2}$.

4. The following data represent the average monthly temperatures in $^\circ F$ in a Southern city.

Month x	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temp.	38.2	42.9	50.6	72.5	80.1	90.1	95.3	98.4	88.2	75.1	56.7	45.9

   Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that models the data.

6.6 mini lecture

$y = A\sin(\omega x - \phi) + B$

$y = A\cos(\omega x - \phi) + B$

Amplitude $= |A|$

period $= T = \frac{2\pi}{\omega}$

ω = omega

ϕ = phi

Phase shift $= \frac{\phi}{\omega}$

B = vertical shift

① $y = -2\cos\left(\frac{x}{2} - \pi\right)$

Amplitude $= |-2| = \boxed{2}$

period $= \frac{2\pi}{\omega} = \frac{2\pi}{1/2} = \boxed{4\pi}$

phase shift $= \frac{\phi}{\omega} = \frac{\pi}{1/2} = \boxed{2\pi}$

6.6 mini lecture continued

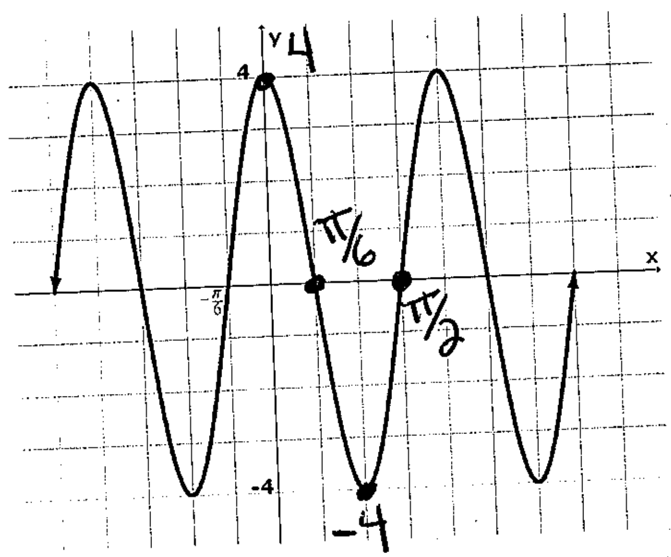
(2) Graph at least 2 periods

$$y = 4\sin\left(3x + \pi/2\right)$$

$$\text{Amplitude} = |4| = \boxed{4}$$

$$\text{Period} = \frac{2\pi}{\omega} = \boxed{\frac{2\pi}{3}}$$

$$\text{Phase shift} = \frac{\phi}{\omega} = \frac{-\pi/2}{3} = \boxed{-\pi/6}$$



6.6 mini lecture continued

③ Sine function with amplitude of $\frac{1}{2}$, period $\pi/4$, phase shift $-\pi/2$

$$y = A\sin(\omega x - \phi) + B$$

$$\text{Amplitude} = \boxed{\frac{1}{2}}$$

$$\omega = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi/4} = \boxed{8}$$

$$\text{Phase shift} = \frac{\phi}{\omega} \rightarrow -\frac{\pi}{2} = \frac{\phi}{8} \quad \boxed{\phi = -4\pi}$$

$$\boxed{y = \frac{1}{2}\sin(8x + 4\pi)}$$

OR

$$\boxed{y = -\frac{1}{2}\sin(8x + 4\pi)}$$

6.6 mini lecture continued

(4) * Calculator

Find sinusoidal function

	1	2	3	4	5	6	7	8	9	10	11	12
Month x	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temp.	38.2	42.9	50.6	72.5	80.1	90.1	95.3	98.4	88.2	75.1	56.7	45.9

L_1 (month numbers), L_2 (temperatures)

$$\boxed{y = 30.11\sin(.49x - 1.99) + 67.74}$$