Mini-Lecture 3.1
Linear Functions and Their Properties

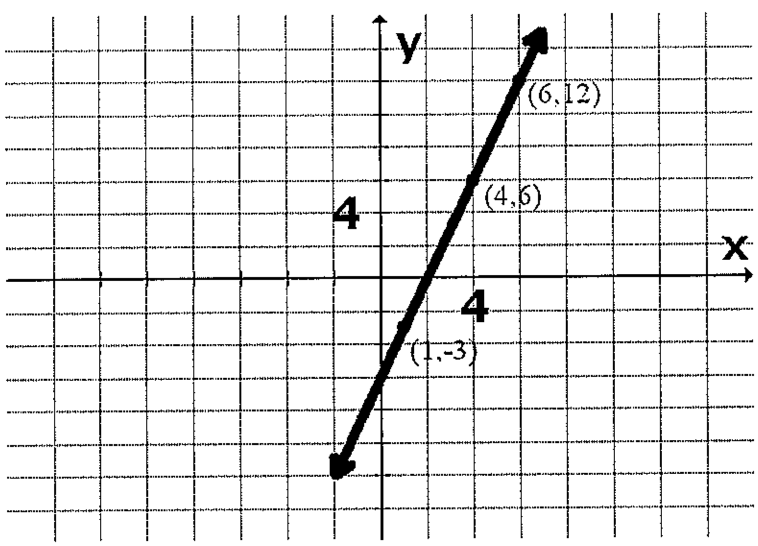
Learning Objectives:

1. Graph Linear Functions (p. 130)
2. Use Average Rate of Change to Identify Linear Functions (p. 130)
3. Determine Whether a Linear function Is Increasing, Decreasing, or Constant (p. 133)
4. Build Linear Models from Verbal Descriptions (p. 134)

Examples:

1. Suppose that $f(x) = 5x - 9$ and $g(x) = -3x + 7$. Solve $f(x) = g(x)$. Then graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution to the equation $f(x) = g(x)$.

2. In parts (a) and (b) using the following figure,



(a) Solve $f(x) = 12$. (b) Solve $0 < f(x) < 12$.

3. The monthly cost C, in dollars, for renting a full-size car for a day from a particular agency is modeled by the function $C(x) = 0.12x + 40$, where x is the number of miles driven. Suppose that your budget for renting a car is \$100. What is the maximum number of miles that you can drive in one day?

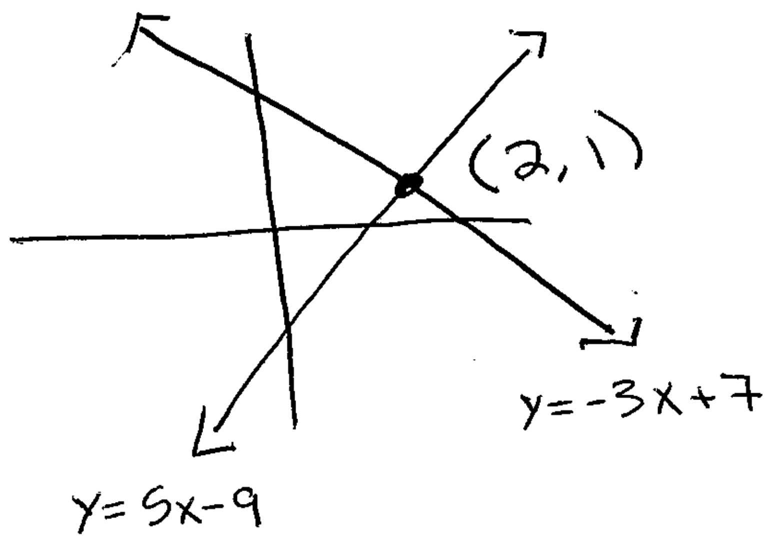
4. Find a firm's break-even point if $R(x) = 10x$ and $C(x) = 7x + 6000$. (Before working this problem, go over the explanation above Problems 43 and 44 on page 290.)

3.1 mini Lecture

① $f(x) = 5x - 9$

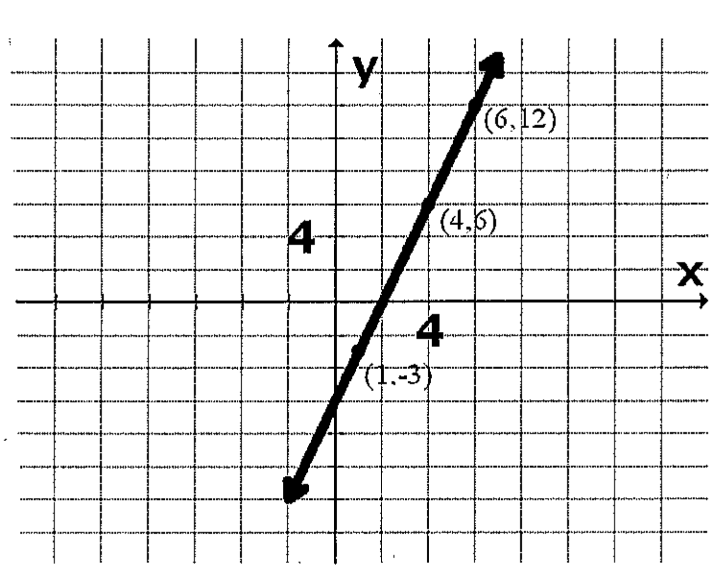
$g(x) = -3x + 7$

$\boxed{(2,1)}$



② Solve $f(x) = 12$ $\boxed{x=6}$ $(6,12)$

Solve $0 < f(x) < 12$ $\boxed{2 < x < 6}$



3.1 mini lecture continued

(3) $C(x) = .12x + 40$

$100 = .12x + 40$

$\boxed{x = 500 \text{ miles}}$

(4) $R(x) = 10x$

$C(x) = 7x + 6000$

Break even

$10x = 7x + 6000$

$\boxed{x = 2000 \text{ units}}$

Mini-Lecture 3.2

Linear Models: Building Linear Models from Data

Learning Objectives:

1. Draw and Interpret Scatter Diagrams (p. 140)
2. Distinguish between Linear and Nonlinear Relations (p. 141)
3. Use a Graphing Utility to Find the Line of Best Fit (p. 142)

Examples:

x	3	7	8	9	11	15
y	2	4	7	8	6	10

1. Draw a scatter diagram. Select two points from the scatter diagram and find the equation of the line containing the two points.

2. Use a graphing utility to find the equation for line of best fit for problem 1.

The marketing manager for a toy company wishes to find a function that relates the demand D for a doll and p the price of the doll. The following data were obtained based on a price history of the doll. The Demand is given in thousands of dolls sold per day.

Price	9.00	10.50	11.00	12.00	12.50	13
Demand	12	11	9	10	9.5	8

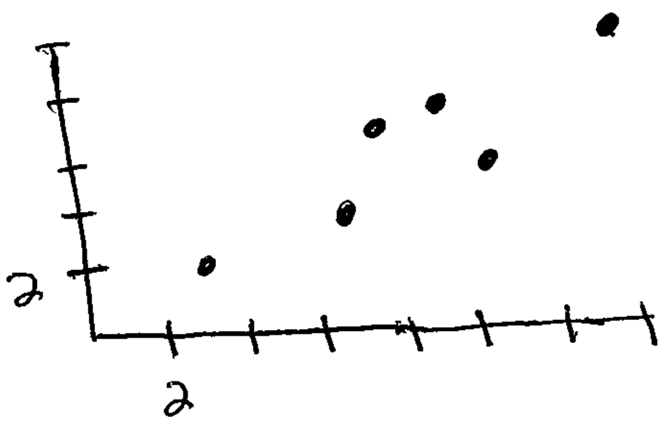
3. Use a graphing utility to draw a scatter diagram. Then, find and draw the line of best fit.

4. How many dolls will be demanded if the price is $11.50?

3.2 mini lecture

①

x	3	7	8	9	11	15
y	2	4	7	8	6	10



If pick
(3,2) and (7,4)

$y = mx + b$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = \dfrac{4-2}{7-3} = \dfrac{1}{2}$

$y = mx + b$

$2 = \frac{1}{2}(3) + b$

$b = \frac{1}{2}$

$\boxed{y = \frac{1}{2}x + \frac{1}{2}}$

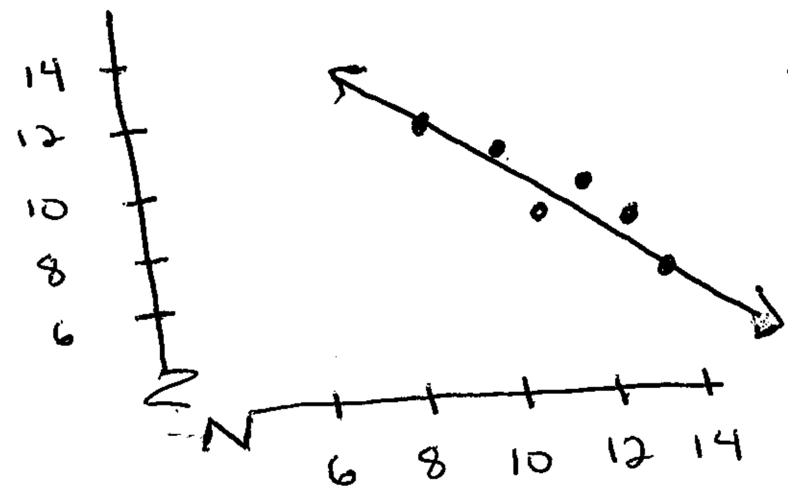
3.2 mini lecture continued

(2) * calculator *

$$\boxed{y = .633x + .575}$$

(3)

Price	9	10.5	11	12	12.5	13
Demand	12	11	9	10	9.5	8



* calculator

$$\boxed{y = -.838x + 19.419}$$

(4) If price = $ 11.50

$y = -.838(11.5) + 19.419$

$\approx \boxed{9.78}$

Mini-Lecture 3.3

Quadratic Functions and Their Properties

Learning Objectives:

1. Graph a Quadratic Function Using Transformations (p. 148)
2. Identify the Vertex and Axis of Symmetry of a Quadratic Function (p. 150)
3. Graph a Quadratic Function Using Its Vertex, Axis, and Intercepts (p. 150)
4. Find a Quadratic Function Given Its Vertex and One Other Point (p. 153)
5. Find the Maximum and Minimum Value of a Quadratic Function (p. 154)

Examples:

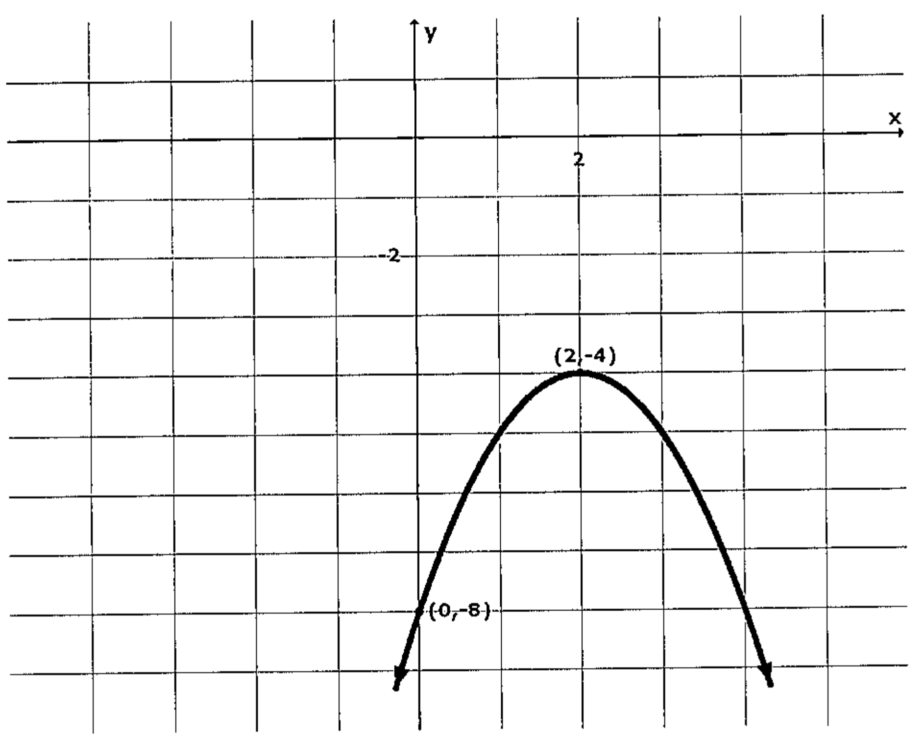
1. Find the coordinates of the vertex for the parabola defined by the given quadratic function. $f(x) = -3x^2 + 5x - 4$

2. Sketch the graph of the quadratic function by determining whether it opens up or down and by finding its vertex, axis of symmetry, y-intercepts, and x-intercepts, if any. $f(x) = 6 - 5x + x^2$

3. For the quadratic function, $f(x) = 4x^2 - 8x$,

   a) determine, without graphing, whether the function has a minimum value or a maximum value,
   b) find the minimum or maximum value.

4. Determine the quadratic function whose graph is given.



3.3 mini lecture

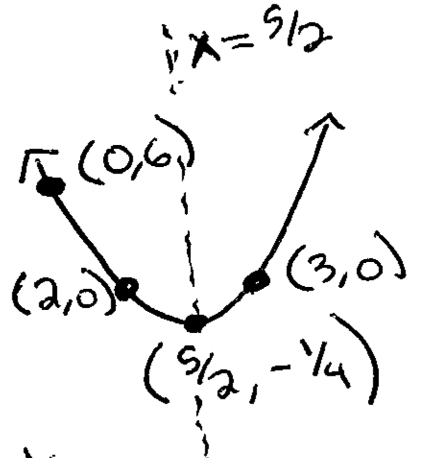
① $f(x) = ax^2 + bx + c$

$f(x) = -3x^2 + 5x - 4$

$* \; x = -\frac{b}{2a} = \frac{-5}{2(-3)} = \boxed{\frac{5}{6}}$

$* \; y = -3\left(\frac{5}{6}\right)^2 + 5\left(5/6\right) - 4 = \boxed{\frac{-23}{12}}$

$\boxed{\text{vertex } \left(\frac{5}{6}, \frac{-23}{12}\right)}$



② $f(x) = 6 - 5x + x^2$

$f(x) = x^2 - 5x + 6$

$* \; x = -\frac{b}{2a} = \frac{5}{2(1)} = \frac{5}{2}$

$* \; y = (5/2)^2 - 5(5/2) + 6 = -1/4$

vertex $(5/2, -1/4)$, opens up, axis symmetry $x = 5/2$

X-int

$0 = x^2 - 5x + 6$

$0 = (x-3)(x-2)$

$\boxed{(2,0), (3,0)}$

Y-int

$y = (0)^2 - 5(0) + 6$

$y = 6$

$\boxed{(0,6)}$

3.3 mini lecture continued

③ $f(x) = 4x^2 - 8x$

a is positive, opens up, $\boxed{\text{minimum value}}$

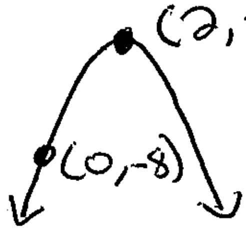
$$x = \frac{-b}{2a} = \frac{-(-8)}{2(4)} = 1$$

$$y = 4(1)^2 - 8(1) = -4$$

minimum value is $\boxed{-4}$

3.3 mini lecture continued

(4)



$$h = -\frac{b}{2a}$$

$$K = \frac{4ac - b^2}{4a}$$

vertex (h, k)

$$f(x) = ax^2 + bx + c$$

$$f(x) = a(x-h)^2 + k$$

$$\overset{y}{-8} = -a(\overset{x}{0} - \overset{h}{2})^2 - \overset{k}{4}$$

$$a = 1$$

$$f(x) = -(x-2)^2 - 4$$

$$\boxed{f(x) = -x^2 + 4x - 8}$$

Mini-Lecture 3.4

Building Quadratic Models from Verbal Descriptions and from Data

Learning Objectives:

1. Build Quadratic Models from Verbal Descriptions (p. 159)
2. Build Quadratic Models from Data (p. 163)

Examples:

1. Among all pairs of numbers whose sum is 50, find a pair whose product is as large as possible. What is the maximum product?

2. A person standing close to the edge of the top of a 180-foot tower throws a ball vertically upward. The quadratic function $s(t) = -16t^2 + 64t + 180$ models the ball's height above ground , $s(t)$, in feet, t seconds after it was thrown. After how many seconds does the ball reach its maximum height? What is the maximum height?

3. The price p (in dollars) and the quantity x sold of a certain product obey the demand equation $p = -\frac{1}{4}x + 120$. Find the model that expresses the revenue R as a function of x. What quantity x maximizes revenue? What is the maximum revenue?

4. The following data represent the percentage of the population in a certain country aged 40 or older whose age is x who do not have a college degree of some type.

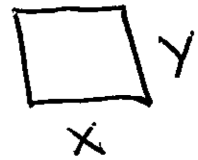
Age, x	40	45	50	55	60	65
No college	25.4	23.2	21.8	24.5	26.1	29.8

Find a quadratic model that describes the relationship between age and percentage of the population that do not have a college degree. Use the model to predict the percentage of 53-year-olds that do not have a college degree.

3.4 mini lecture

① $x + y = 50$

$A = xy$



$y = 50 - x$

$A = x(50 - x)$

$A = -x^2 + 50x$

$* \; x = \frac{-b}{2a} = \frac{-50}{2(-1)} = \underline{x = 25}$

$* \; x + y = 50$

$25 + y = 50$

$\underline{y = 25}$

$\boxed{(25, 25)}$

$\boxed{A = 625}$

② $s(t) = -16t^2 + 64t + 180$

$*$ vertex $\quad x = \frac{-b}{2a} = \frac{-64}{2(-16)} = \boxed{2 \text{ sec}}$

$*$ max $\quad y = -16(2)^2 + 64(2) + 180$

$= \boxed{244 \text{ ft}}$

3.4 mini lecture continued

③ $p = -\frac{1}{4}x + 120$

$R(x) = xp$

$R(x) = x(-1/4\,x + 120)$

$R(x) = -\frac{1}{4}x^2 + 120x$

* $x = \frac{-b}{2a} = \frac{-120}{2(-1/4)} = \boxed{240 \text{ units}}$

* $y = -\frac{1}{4}(240)^2 + 120(240) = \boxed{\$14,400}$

④

AGE	40	45	50	55	60	65
no college	25.4	23.2	21.8	24.5	26.1	29.8

* calculator

$$\boxed{y = .0296x^2 - 2.9216x + 94.655}$$

$$y = .0296(53)^2 - 2.9216(53) + 94.655$$

$$= \boxed{23.1\%}$$

Mini-Lecture 3.5

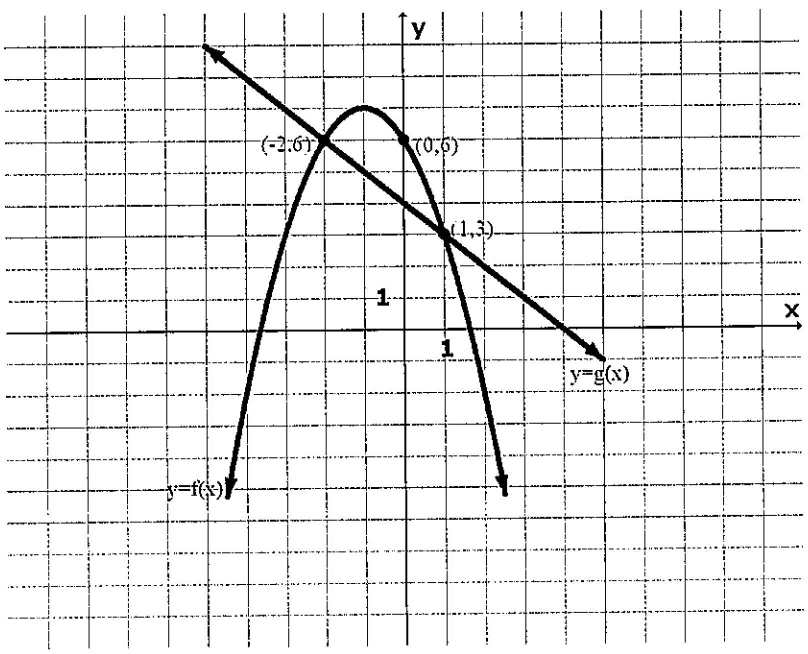
Inequalities Involving Quadratic Functions

Learning Objectives:

1. Solve Inequalities Involving a Quadratic Function (p. 169)

Examples:

1. Use the figure to solve the inequality $f(x) \geq g(x)$.

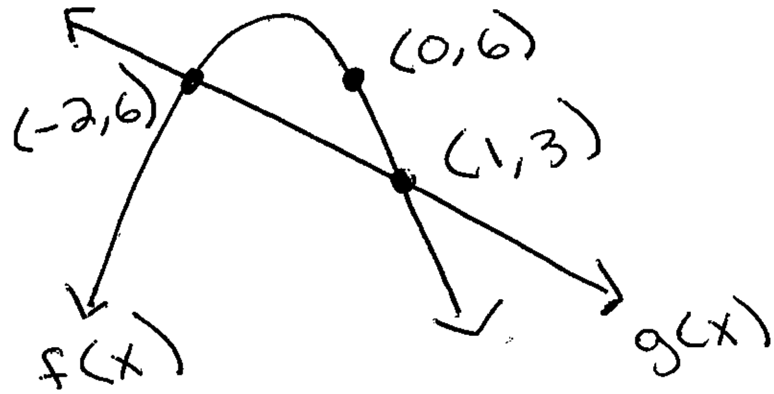


2. Solve and express the solution in interval notation. $9x^2 - 6x + 1 < 0$

3. Solve the inequality. $x(x+2) > 15$

4. Solve $f(x) > g(x)$. $f(x) = -x^2 + 2x + 3$; $g(x) = -x^2 - 2x + 8$

mini Lecture 3.5

① 

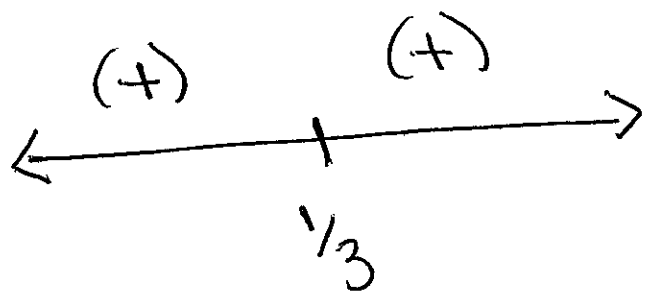
$f(x) \geq g(x)$

$\boxed{[-2, 1]}$

② $9x^2 - 6x + 1 < 0$

$(3x-1)(3x-1) < 0$

critical points $x = 1/3$



* pick any value to the left of 1/3 and to the right of 1/3. Check to see if the value is (+) or (-)

$\boxed{\emptyset}$

no value is less than 0 (-) so [redacted] no solution exists

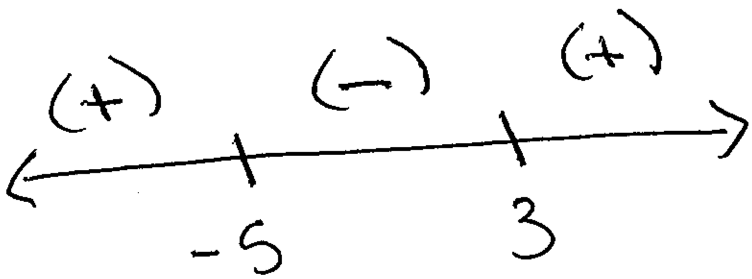
3.5 mini lecture continued

③ $x(x+2) > 15$

$x^2 + 2x - 15 > 0$

$(x+5)(x-3) > 0$

critical points $x = -5, 3$



Since >0 is looking for (+) number

$$\boxed{(-\infty, -5) \cup (3, \infty)}$$

mini lecture 3.5 continued

4. $f(x) = -x^2 + 2x + 3$

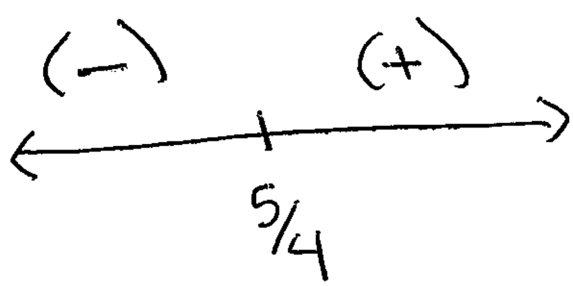
$g(x) = -x^2 - 2x + 8$

$f(x) > g(x)$

$-x^2 + 2x + 3 > -x^2 - 2x + 8$

$4x - 5 > 0$

critical point $x = 5/4$



$\boxed{(5/4, \infty)}$