Mini-Lecture 2.1

Functions

Learning Objectives:

1. Determine Whether a Relation Represents a Function (p. 58)
2. Find the Value of a Function (p. 61)
3. Find the Domain of a Function Defined by an Equation (p. 64)
4. Form the Sum, Difference, Product, and Quotient of Two Functions (p. 66)

Examples:

1. Determine whether $y^2 = x+1$ defines y as a function of x.

2. Find the domain of $f(x) = \dfrac{1}{x-2} - \dfrac{1}{x+3}$.

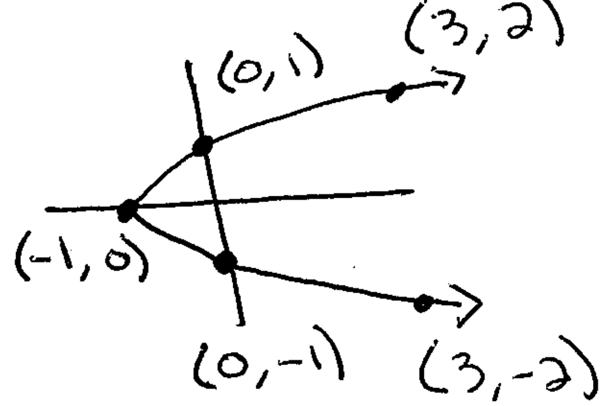
3. $f(x) = \dfrac{3x+2}{x^2-16}$ and $g(x) = \dfrac{5x-4}{x^2-16}$, find $f+g$, $f-g$, fg, $\dfrac{f}{g}$. Determine the domain of each.

4. Find and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$, $h \neq 0$ for $f(x) = -2x^2 - x + 3$.

mini Lecture 2.1

① $y^2 = x+1$

$y = \pm\sqrt{x+1}$



* vertical line test

$\boxed{\text{not a function}}$

* Function maps one x to one y value

② $f(x) = \frac{1}{x-2} - \frac{1}{x+3}$

$x-2 \neq 0$ $\quad$ $x+3 \neq 0$

$x \neq 2$ $\quad$ $x \neq -3$

$\boxed{(-\infty,-3)\cup(-3,2)\cup(2,\infty)}$

Domain

Mini Lecture
2.1 continued

3 $f(x) = \frac{3x+2}{x^2-16}$ $g(x) = \frac{5x-4}{x^2-16}$

a) $f+g$ $\frac{3x+2}{x^2-16} + \frac{5x-4}{x^2-16} = \boxed{\frac{8x-2}{x^2-16}}$

Domain: $(-\infty,-4)\cup(-4,4)\cup(4,\infty)$

b) $f-g$ $\frac{3x+2}{x^2-16} - \frac{5x-4}{x^2-16} = \boxed{\frac{-2x+6}{x^2-16}}$

Domain: $(-\infty,-4)\cup(-4,4)\cup(4,\infty)$

c) fg $\frac{3x+2}{x^2-16} \cdot \frac{5x-4}{x^2-16} = \boxed{\frac{15x^2-2x-8}{x^4-32x^2+256}}$

Domain: $(-\infty,-4)\cup(-4,4)\cup(4,\infty)$

d) $\frac{f}{g}$ $\frac{3x+2}{x^2-16} \cdot \frac{x^2-16}{5x-4} = \boxed{\frac{3x+2}{5x-4}}$

Domain: $(-\infty,-4)\cup(-4,\frac{4}{5})\cup(\frac{4}{5},4)\cup(4,\infty)$

Mini Lecture
2.1 continued

(4) $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$

$f(x) = -2x^2 - x + 3$

$$\frac{\left[-2(x+h)^2-(x+h)+3\right]-\left[-2x^2-x+3\right]}{h}$$

$$\frac{\cancel{-2x^2}-4xh-2h^2-x-h+3+2x^2+x-3}{h}$$

$$\frac{-4xh-2h^2-h}{h}$$

$$\boxed{-4x-2h-1}$$

Mini-Lecture 2.2

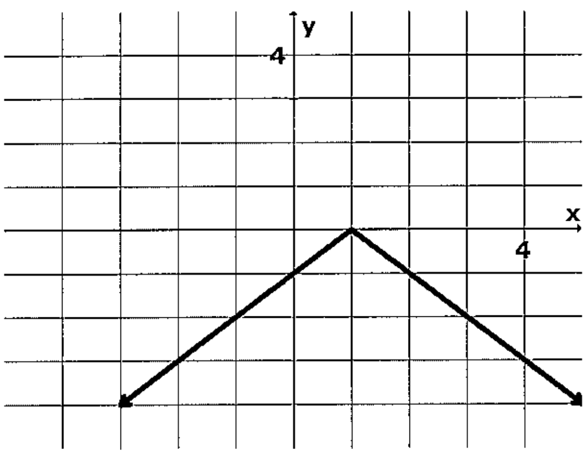
The Graph of a Function

Learning Objectives:

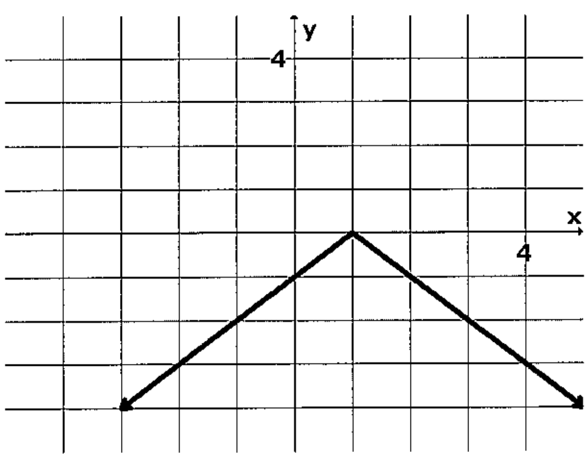
1. Identify the Graph of a Function (p. 72)
2. Obtain Information from or about the Graph of a Function (p. 73)

Examples:

1. Determine whether the graph is that of a function by using the vertical line test.

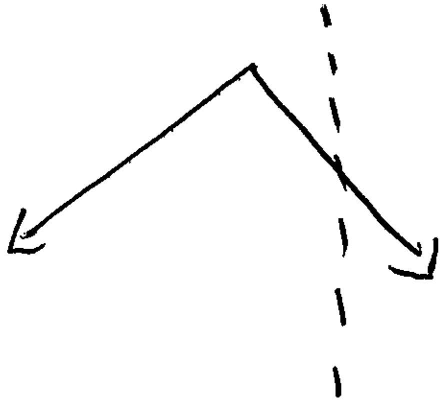


Use the graph to determine 2) the function's domain and the function's range, 3) x-intercepts, if any and y-intercepts, if any, 4) If $x = -2$, what is $f(x)$? What is the point on the graph?



mini Lecture 2.2

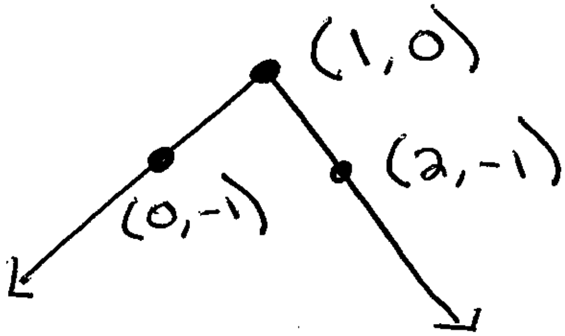
(1)



* vertical line test
if it hits one point (at most)
it is a function,
more than one point
it is not a function

$\boxed{\text{Function}}$

(2)



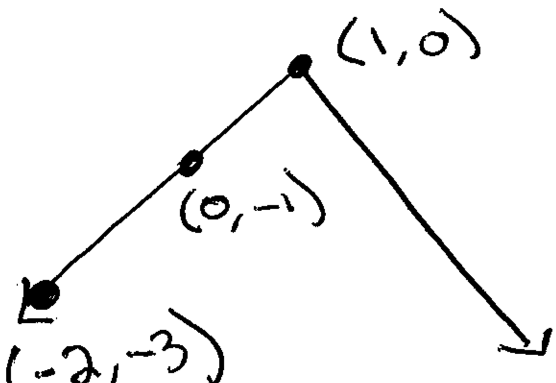
Domain: $(-\infty, \infty)$

Range: $(-\infty, 0]$

(3) Figure in #2

$\boxed{\begin{array}{l}\text{X-intercept } (1,0)\\ \text{Y-intercept } (0,-1)\end{array}}$

(4)



If $x=-2$, then

$y=-3$

$\boxed{(-2,-3)}$

$\boxed{f(-2)=-3}$

Mini-Lecture 2.3

Properties of Functions

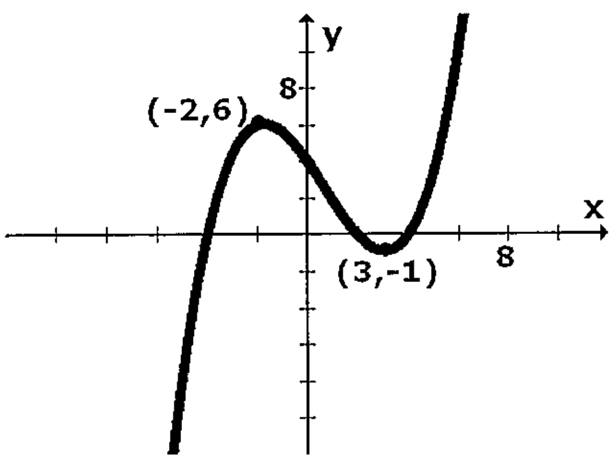
Learning Objectives:

1. Determine Even and Odd Functions from a Graph (p. 81)
2. Identify Even and Odd Functions from the Equation (p. 82)
3. Use a Graph to Determine Where a Function Is Increasing, Decreasing or Constant (p. 83)
4. Use a Graph to Locate Local Maxima and Local Minima (p. 84)
5. Use a Graph to Locate the Absolute Maximum and Minimum (p. 85)
5. Use a Graphing Utility to Approximate Local Maxima and Local Minima and to Determine Where a Function Is Increasing or Decreasing (p. 86)
6. Find the Average Rate of Change of a Function (p. 87)

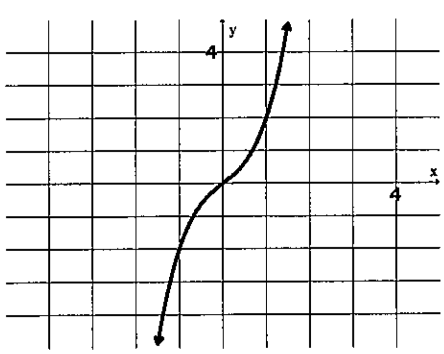
Examples:

1. Determine whether $f(x) = 3x^3 - 5x^7$ is even or odd or neither.

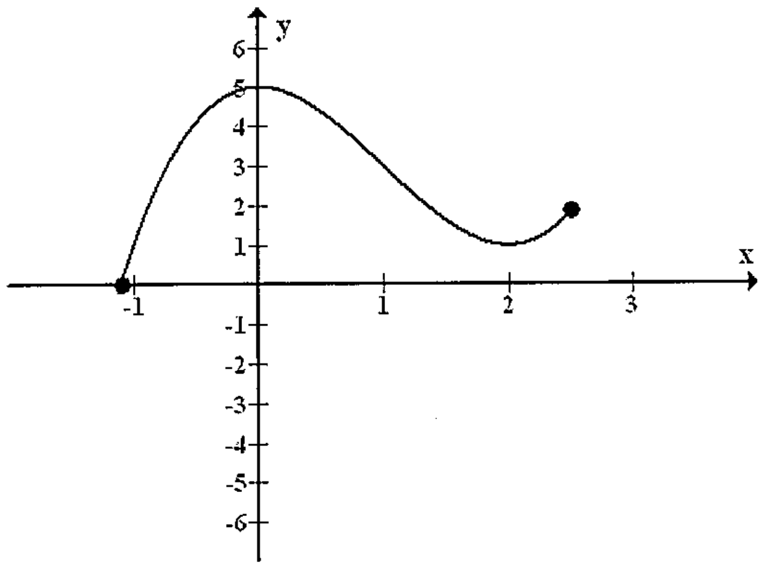
2. Give the intervals on which the function is increasing, decreasing, or constant.



3. Find the relative maxima and relative minima of the graph below.



4. Find the average rate of change of $f(x) = x^2 - x + 4$ from $x_1 = 2$ to $x_2 = 6$.
5. For the graph of f(x) below, find the absolute maximum and absolute minimum, if they exist.



mini Lecture 2.3

① odd, even, neither $f(x) = 3x^3 - 5x^7$

* odd symmetric with origin

$$f(-x) = -f(x)$$

* even symmetric with y-axis

$$f(-x) = f(x)$$

odd

$$3(-x)^3 - 5(-x)^7 = -(3x^3 - 5x^7)$$

$$-3x^3 + 5x^7 = -3x^3 + 5x^7 \quad \checkmark$$

same

$\boxed{\text{odd}}$

even

$$3(-x)^3 - 5(-x)^7 = 3x^3 - 5x^7$$

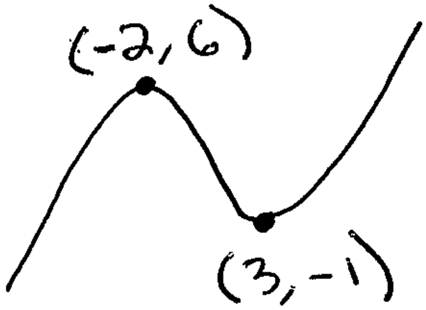
$$-3x^3 + 5x^7 = 3x^3 - 5x^7$$

different

not even

mini Lecture 2.3 continued

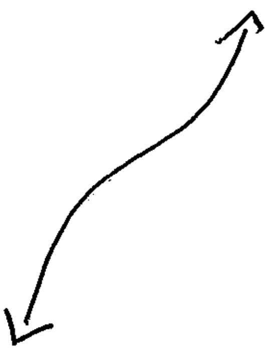
2



Increasing
$(-\infty, -2) \cup (3, \infty)$

Decreasing
$(-2, 3)$

3



no relative maxima
no relative minima

4 Average Rate of Change

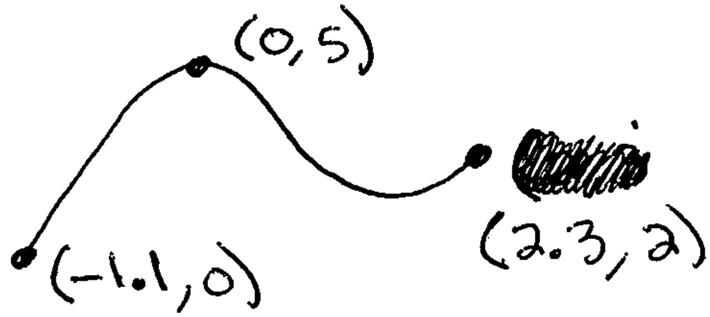
$f(x) = x^2 - x + 4$ from 2 to 6

$$\frac{f(b) - f(a)}{b - a} = \frac{\left[(6)^2 - (6) + 4\right] - \left[(2)^2 - (2) + 4\right]}{6 - 2}$$

$$= \frac{34 - 6}{4} = \boxed{7}$$

mini Lecture 2.3 continued

5



Absolute max $\boxed{f(0)=5}$

Absolute min $\boxed{f(-1.1)=0}$

Mini-Lecture 2.4

Library of Functions; Piecewise-defined Functions

Learning Objectives:

1. Graph the Functions Listed in the Library of Functions (p. 93)
2. Graph Piecewise-defined Functions (p. 98)

Examples:

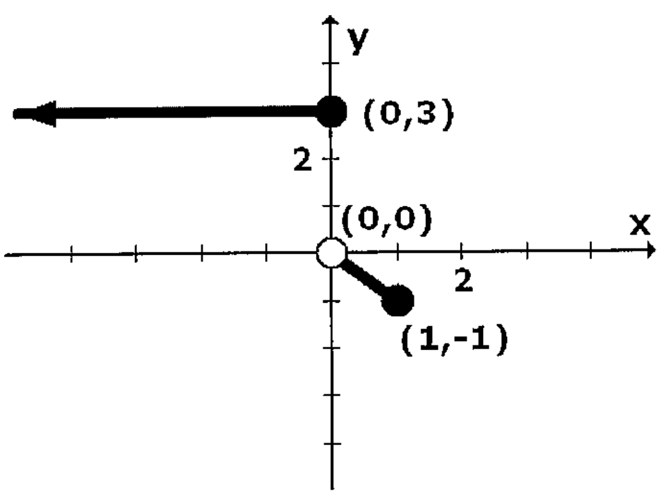
1. Find $h(-5)$, if

$$h(x)=\begin{cases} x+4 & if \quad x \geq -4 \\ -(x+4) & if \quad x<-4 \end{cases}$$

2. Graph f. $$f(x)=\begin{cases} |x| & if \ -3 \leq x<1 \\ \sqrt[3]{x} & if \ x>1 \end{cases}$$

3. Find the domain and the range of the function in Problem 2.

4. Write a definition for the following piecewise-defined function.

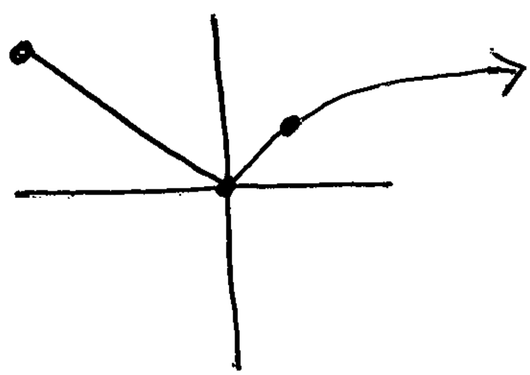


2.4 mini lecture

① find $h(-5)$ if $h(x)=\begin{cases} x+4 & \text{if } x \geq -4 \\ -(x+4) & \text{if } x < -4 \end{cases}$

$$\begin{aligned} h(-5) &= -(x+4) \\ &= -(-5+4) \\ &= \boxed{1} \end{aligned}$$

② Graph $f(x)=\begin{cases} |x| & \text{if } -3 \leq x < 1 \\ \sqrt[3]{x} & \text{if } x > 1 \end{cases}$

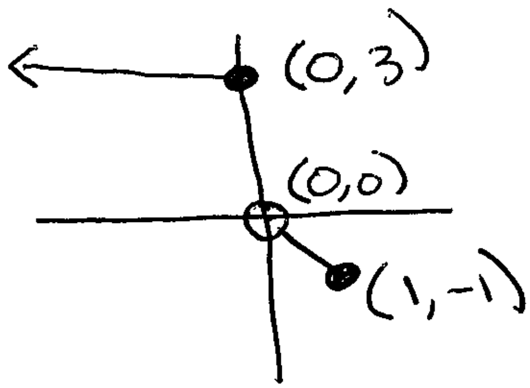


③ Domain $[-3, \infty)$

Range $[0, \infty)$

Mini Lecture
2.4 continued

4) write a definition



$$f(x) = \begin{cases} 3 & \text{if } x \leq 0 \\ -x & \text{if } 0 < x \leq 1 \end{cases}$$

Mini-Lecture 2.5

Graphing Techniques: Transformations

Learning Objectives:

1. Graph Functions Using Vertical and Horizontal Shifts (p. 104)
2. Graph Functions Using Compressions and Stretches (p. 106)
3. Graph Functions Using Reflections about the x-Axis or y-Axis (p. 108)

Examples:

1. Find the equation of the function that is finally graphed after the following transformations are applied to the graph of $y = x^2$.

    a. Shift right 3 units.
    b. Reflect about the x-axis.
    c. Shift down 2 units.

2. Suppose that the x-intercepts of the graph of $y = f(x)$ are 2 and -4. What are the x-intercepts of $2f(x)$.

3. Begin by graphing $f(x) = \sqrt{x}$. Then use transformations to graph $f(x) = \frac{1}{2}\sqrt{x} - 2$.

4. Begin by graphing $f(x) = x^3$. Then use transformations to graph $f(x) = -(x+1)^3 - 3$

Mini Lecture 2.5

① $y = x^2$

a) Shift right 3 units
b) Reflect about x-axis
c) Shift down 2 units

$$\boxed{y = -(x-3)^2 - 2}$$

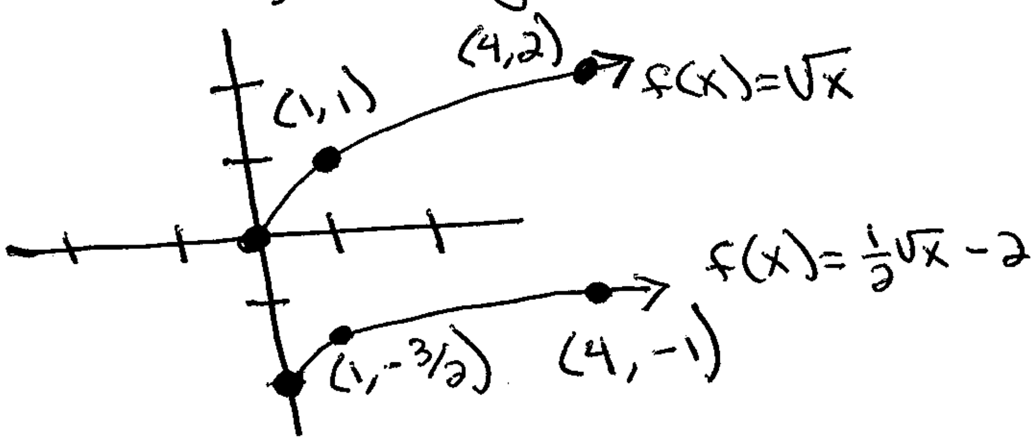
② x-intercepts of $y = f(x)$ are $2, -4$

so, x-intercepts of

$y = 2f(x)$ are $\boxed{2, -4}$

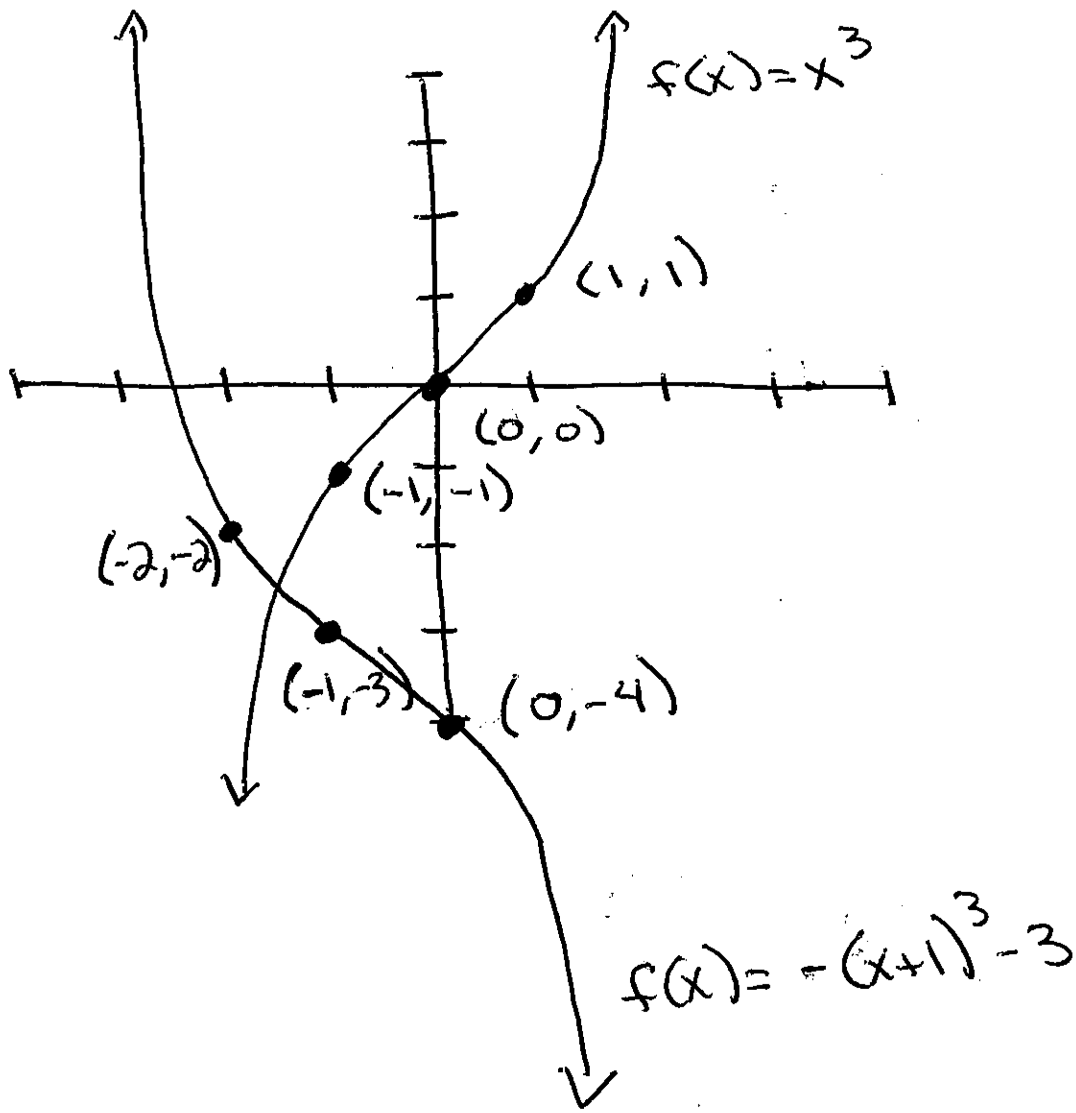
* multiply does not change intercepts

③ Graph $f(x) = \sqrt{x}$, then graph $f(x) = \frac{1}{2}\sqrt{x} - 2$



mini lecture 2.5 continued

④ Graph $f(x) = x^3$, then graph $f(x) = -(x+1)^3 - 3$



Mini-Lecture 2.6

Mathematical Models; Building Functions

Learning Objectives:

1. Build and Analyze Functions (p.116)

Examples:

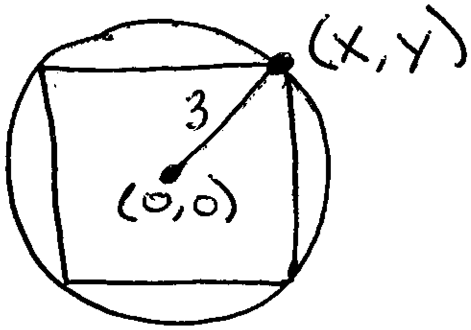
1. A rectangle is inscribed in a circle of radius 3. Let $P=(x,y)$ be a point in quadrant I that is a vertex of the rectangle and is on the circle. Express the area A of the rectangle as a function of x and the perimeter P of the rectangle as a function of x.

2. Let $P=(x,y)$ be a point on the graph of $y=x^3$. Express the distance d from P to the point (2, 0) as function of x. What is d if $x=-1$?

3. A right triangle has one vertex on the graph of $y=16-x^2$, $x>0$, at (x,y), another at the origin, and the third on the positive x-axis at $(x,0)$. Express the area A of the triangle as a function of x.

4. An open box with a square base is to be made from a square piece of cardboard 16 inches on a side by cutting out a square from each corner and turning up the sides. Express the volume V of the box as a function of the length x of the side of the square cut from each corner. Find the volume if a 2-inch square is cut out.

mini Lecture 2.6

① 

$A = LW$

$A = (2x)(2y)$

$A = 4xy$

$x^2 + y^2 = r^2$

$x^2 + y^2 = 3^2$

$x^2 + y^2 = 9$

$y = \sqrt{9 - x^2}$

$\boxed{A = 4x(\sqrt{9 - x^2})}$

$P = 4x + 4y$

$\boxed{P = 4x + 4(\sqrt{9 - x^2})}$

② $y = x^3$ $\quad P(x, y)$ to $(2, 0)$

$d = \sqrt{(x-2)^2 + (y-0)^2}$

$d = \sqrt{x^2 - 4x + 4 + y^2}$

$y = x^3$

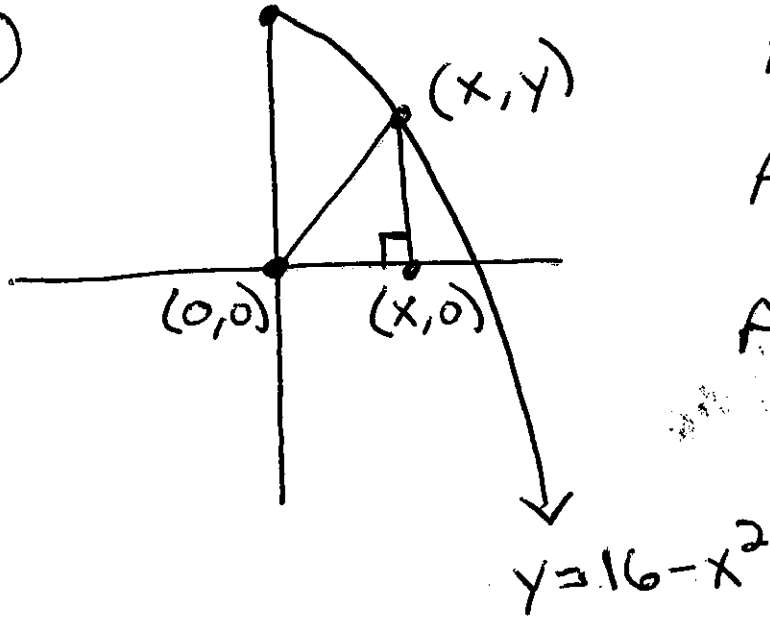
$y^2 = x^6$

$\boxed{d = \sqrt{x^6 + x^2 - 4x + 4}}$

$x = -1$ $\quad d = \sqrt{(-1)^6 + (-1)^2 - 4(-1) + 4}$ $\quad \boxed{d = \sqrt{10}}$

mini Lecture 2.6 continued

③



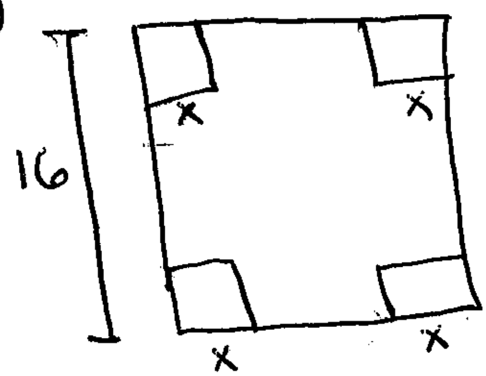
$A = \frac{1}{2}bh$

$A = \frac{1}{2}(x)(y)$

$A = \frac{1}{2}(x)(16-x^2)$

$\boxed{A = 8x - \frac{1}{2}x^3}$

④



$V = LWH$

$V = (16-2x)(16-2x)(x)$

$\boxed{V = 4x(8-x)^2}$

If 2 inch square cut out

$V = 4(2)(8-2)^2 = \boxed{288 \text{ in}^3}$

Mini-Lecture 5.2

One-to One Functions; Inverse Functions

Learning Objectives:

1. Determine Whether a Function is One-to-One (p. 259)
2. Determine the Inverse of a Function Defined by a Map or a Set of Ordered Pairs (p. 261)
3. Obtain the Graph of the Inverse Function from the Graph of the Function (p. 263)
4. Find the Inverse of a Function Defined by an Equation (p. 264)

Examples:

1. Find the inverse of the one-to-one function. State the domain and range of the inverse.
   $\{(-2,-4),(1,0),(6,-3),(2,4),(-3,-5)\}$

2. $f(x)=\frac{1}{2}x-4$, find $f^{-1}(x)$.

3. $f(x)=\frac{2x-3}{x+1}$, find $f^{-1}(x)$.

4. Using interval notation give the domain and range of f and f^{-1}, if $f(x)=x^3-8$.

mini Lecture 5.2

① Find Inverse. State Domain and Range of the Inverse.

$\{(-2,-4),(1,0),(6,-3),(2,4),(-3,-5)\}$

* flip x & y

$\boxed{\{(-4,-2),(0,1),(-3,6),(4,2),(-5,-3)\}}$

$\boxed{\text{Domain } \{-5,-4,-3,0,4\}}$

one to one (horizontal line hits at most 1 point)
*

② $f(x)=\frac{1}{2}x-4$ find $f^{-1}(x)$

* flip x & y

$x=\frac{1}{2}y-4$

$x+4=\frac{1}{2}y$

$y=2x+8$

$\boxed{f^{-1}(x)=2x+8}$

mini lecture
5.2 continued

③ $f(x) = \frac{2x-3}{x+1}$ find $f^{-1}(x)$

* flip x and y

$$x = \frac{2y-3}{y+1}$$

$$x(y+1) = 2y-3$$

$$xy + x = 2y - 3$$

$$xy - 2y = -x - 3$$

$$y(x-2) = -x-3$$

$$y = \frac{-x-3}{(x-2)}$$

$$y = -\frac{(x+3)}{x-2}$$

$$y = \frac{x+3}{2-x}$$

$$\boxed{f^{-1}(x) = \frac{x+3}{2-x}}$$

Mini Lecture

5.2 continued

(4) Interval notation

give domain and range

of f and f^{-1}

If $f(x) = x^3 - 8$

$f(x) = x^3 - 8$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

$f^{-1}(x)$

$x = y^3 - 8$

$x + 8 = y^3$

$y = \sqrt[3]{x+8}$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

* Domain of f = Range of f^{-1}

* Range of f = Domain of f^{-1}