Mini-Lecture 1.1

The Distance and Midpoint Formulas; Graphing Utilities; Introduction to Graphing Equations

Learning Objectives:

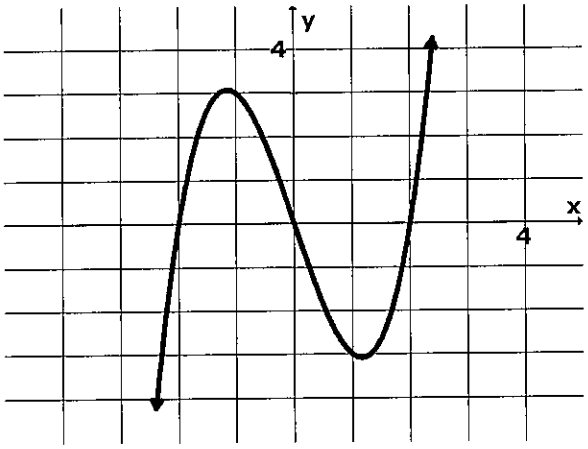
1. Use the Distance Formula (p. 4)
2. Use the Midpoint Formula (p. 7)
3. Graph Equations by Hand by Plotting Points (p. 7)
4. Graph Equations Using a Graphing Utility (p. 10)
5. Use a Graphing Utility to Create Tables (p. 12)
6. Find Intercepts from a Graph (p. 12)
7. Use a Graphing Utility to Approximate Intercepts (p. 13)

Examples:

1. Determine the distance $d(P_1, P_2)$ between the points $P_1(1.4, 3.2)$ and $P_2(-.4, 1.7)$.

2. Find the midpoint of the line segment joining the points $P_1(1.4, 3.2)$ and $P_2(-.4, 1.7)$.

3. The graph of an equation is given. List the intercepts of the graph.



4. Graph $2x - 3y = 56$ using a graphing utility.

mini Lecture 1.1

① $P_1(1.4, 3.2)$ $P_2(-.4, 1.7)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-.4 - 1.4)^2 + (1.7 - 3.2)^2}$$

$$\boxed{d = 2.34}$$

② $P_1(1.4, 3.2)$ $P_2(-.4, 1.7)$

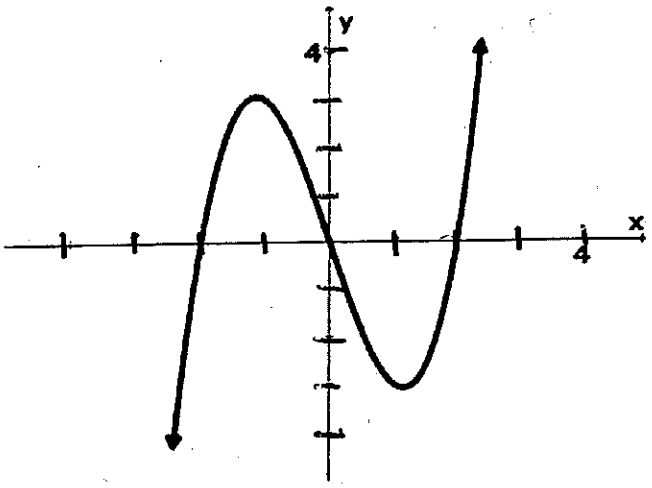
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$m = \left(\frac{1.4 + (-.4)}{2}, \frac{3.2 + 1.7}{2} \right)$$

$$m = \left(\tfrac{1}{2}, \frac{4.9}{2} \right)$$

$$\boxed{m = (.5, 2.45)}$$

③ List intercepts from Graph



x-int

$(-2,0), (0,0), (2,0)$

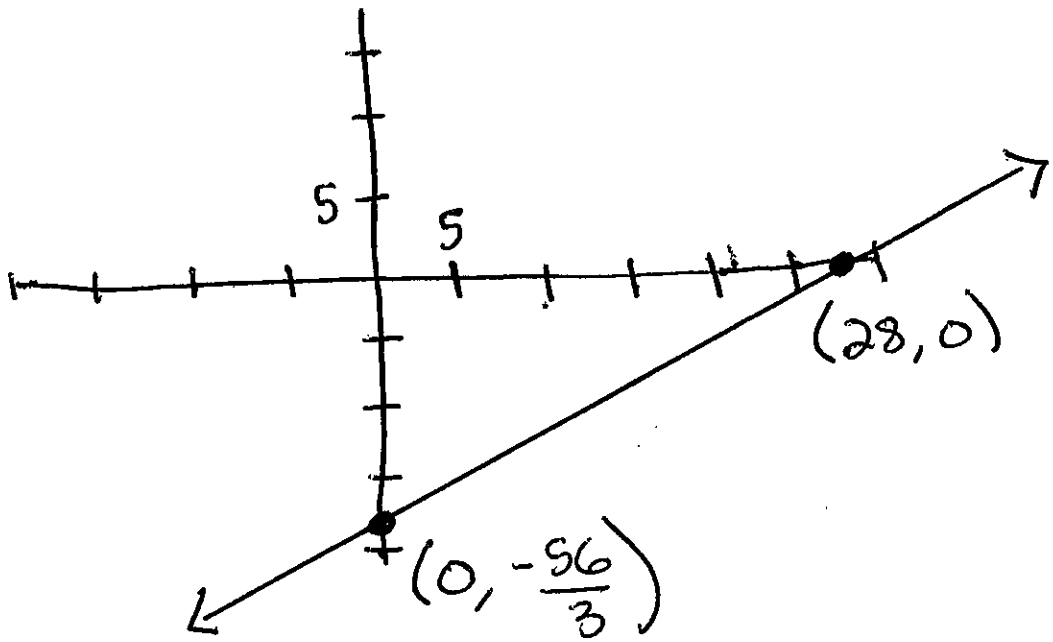
y-int

$(0,0)$

④ Graph $2x-3y=56$

$$-3y = -2x + 56$$

$$y = \frac{2}{3}x - \frac{56}{3}$$



Mini-Lecture 1.2

Intercepts; Symmetry; Graphing Key Equations

Learning Objectives:

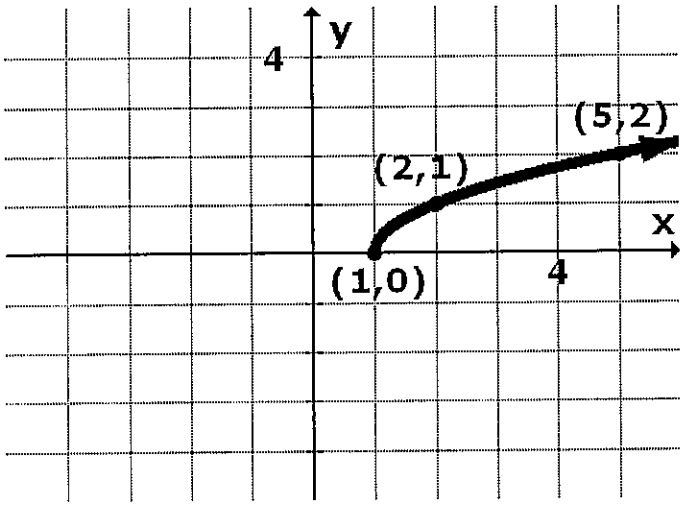
1. Find Intercepts Algebraically from an Equation (p. 18)
2. Test an Equation for Symmetry (p. 19)
3. Know How to Graph Key Equations (p. 21)

Examples:

1. Find the intercepts and graph the equation by plotting points. $6x-3y=-12$

2. Test for symmetry. $f(x)=3x^3-5x^7$

3. Draw a complete graph so that it has x-axis symmetry.



4. Find the intercepts, test for symmetry with respect to the x-axis, y-axis or origin, and graph by hand $y=x^3-8$. Verify the results using a graphing utility.

mini Lecture 1.2

① Find Intercepts $6x-3y=-12$

$-3y=-6x-12$

$y=2x+4$

Find x-int

$(0)=2x+4$

$x=-2$

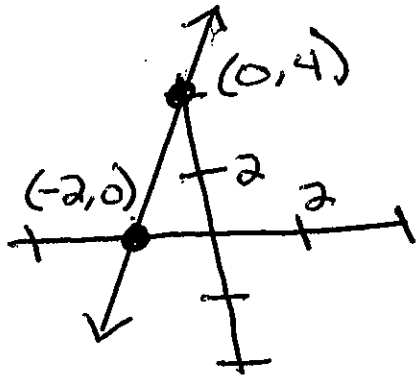
$(-2,0)$

Find y-int

$y=2(0)+4$

$y=4$

$(0,4)$



② Test for symmetry $f(x)=3x^3-5x^7$

Symmetry x-axis

let $y=-y$

$-y=3x^3-5x^7$

$y=-3x^3+5x^7$

different

no

Symmetry y-axis

let $x=-x$

$y=3(-x)^3-5(-x)^7$

$y=-3x^3+5x^7$

different

no

Symmetry origin

let $x=-x$

let $y=-y$

$-y=3(-x)^3-5(-x)^7$

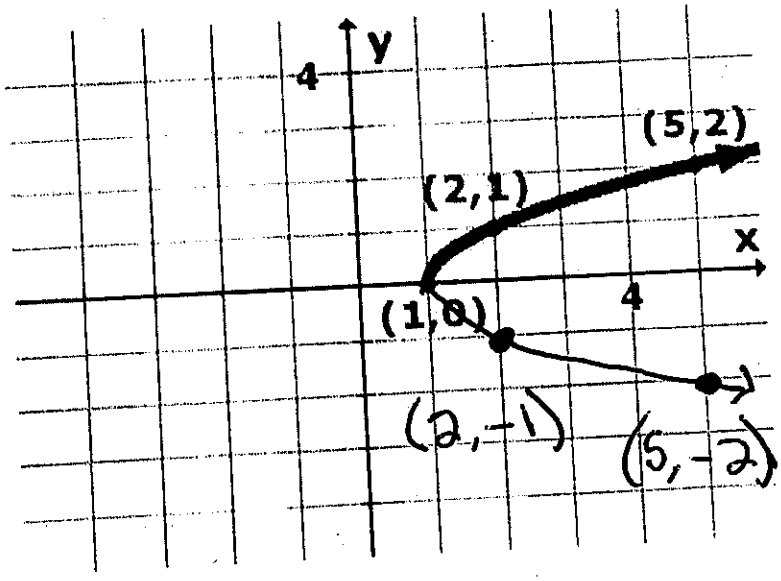
$-y=-3x^3+5x^7$

$y=3x^3-5x^7$

same

yes

③ Complete for x-axis symmetry



④ Find Intercepts, test for symmetry, graph

$y = x^3 - 8$

x-int

$0 = x^3 - 8$

$x^3 = 8$

$x = 2$

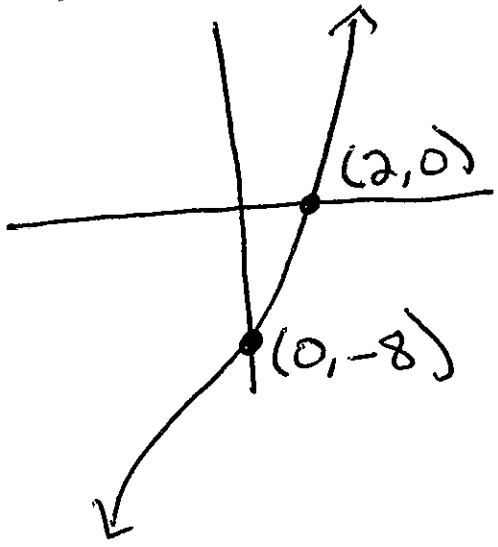
$(2,0)$

y-int

$y = (0)^3 - 8$

$y = -8$

$(0,-8)$



Symmetry x-axis

$-y = x^3 - 8$

$y = -x^3 + 8$

no

Symmetry y-axis

$y = (-x)^3 - 8$

$y = -x^3 - 8$

no

Symmetry origin

$-y = (-x)^3 - 8$

$-y = -x^3 - 8$

$y = x^3 + 8$

no

Mini-Lecture 1.3

Solving Equations Using a Graphing Utility

Learning Objectives:

1. Solve Equations Using a Graphing Utility (p. 26)

Examples:

Use a graphing utility to approximate the real solutions, if any, of each equation rounded to two decimal places. All solutions lie between -10 and 10.

1. $x^3 + 2 = 3x$

2. $x^4 + 3x^2 - x - 5 = 0$

Solve each equation algebraically. Verify your results using a graphing utility.

3. $16x + 2 = 13x - 7$

4. $\frac{-1}{x+1} = \frac{1}{3x+3} - \frac{2}{x-4}$

mini Lecture 1.3

① Solve using calculator

$x^3 + 2 = 3x$

$x^3 - 3x + 2 = 0$

* $Y_1 = x^3 - 3x + 2$

$Y_2 = 0$

$x = 1$

$x = -2$

② Solve using calculator

$x^4 + 3x^2 - x - 5 = 0$

* $Y_1 = x^4 + 3x^2 - x - 5$

$Y_2 = 0$

$x = -1$

$x = 1.19$

③ Solve

$16x + 2 = 13x - 7$

$3x = -9$

$x = -3$

④ Solve

$$-\frac{1}{x+1} = \frac{1}{3x+3} - \frac{2}{x-4}$$

$$-\frac{1}{x+1} = \frac{1}{3(x+1)} - \frac{2}{x-4}$$

$$-\frac{1}{x+1}\left((3)(x+1)(x-4)\right) = \frac{1}{3(x+1)}\left((3)(x+1)(x-4)\right) - \frac{2}{x-4}\left((3)(x+1)(x-4)\right)$$

$$-1(3)(x-4) = 1(x-4) - 2(3)(x+1)$$

$$-3x+12 = x-4-6x$$

$$-6$$

$$2x = -22$$

$$\boxed{x = -11}$$

Mini-Lecture 1.4

Lines

Learning Objectives:

1. Calculate and Interpret the Slope of a Line (p. 29)
2. Graph Lines Given a Point and the Slope (p. 32)
3. Find the Equation of a Vertical Line (p. 32)
4. Use the Point-Slope Form of a Line; Identify Horizontal Lines (p. 33)
5. Find the Equation of a Line Given Two Points (p. 34)
6. Write the Equation of a Line in Slope-Intercept Form (p. 34)
7. Identify the Slope and y-Intercept of a Line from Its Equation (p. 35)
8. Graph Lines Written in General Form Using Intercepts (p. 36)
9. Find Equations of Parallel Lines (p. 37)
10. Find Equations of Perpendicular Lines (p. 38)

Examples:

1. Write the slope-intercept form of the line passing through (-1, 5) with x-intercept -4.

2. Give the slope and y-intercept of $6x-12=0$. Graph the line.

3. Graph the line containing $P(-1,3)$ and having slope $m=-\frac{1}{3}$.

   Also, write the point-slope form of the line.

4. Write the equation of the line in general form through (-5, -3) and perpendicular to $3x-y+4=0$.

mini Lecture 1.4

① Slope-intercept form $(-1, 5)$ with x-int -4.

$(-4, 0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-0}{-1-(-4)} = \frac{5}{3} \quad \text{slope}$$

$y = mx + b$

$0 = \frac{5}{3}(-4) + b$

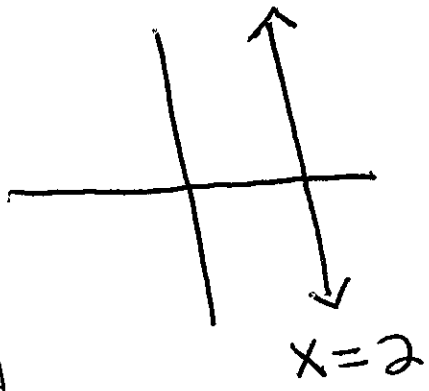
$b = \frac{20}{3}$

$$y = \frac{5}{3}x + \frac{20}{3}$$

② Find slope and y-intercept

$6x - 12 = 0$

$x = 2$



m = undefined

b = no y-intercept

* vertical line $x = a$

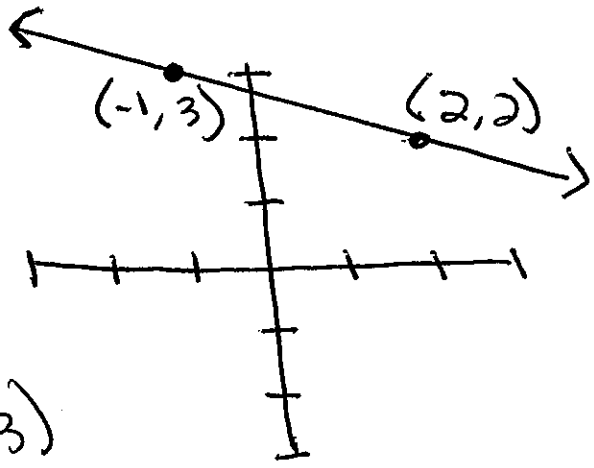
* horizontal line $y = b$

Mini lecture
1.4 continued

(3) Graph $P(-1,3)$ with slope $m = -\frac{1}{3}$

$y - y_1 = m(x - x_1)$

$\boxed{y - 3 = -\frac{1}{3}(x+1)}$



(4) General form $(-5,-3)$ perpendicular to $3x - y + 4 = 0$

$3x - y + 4 = 0$

$-y = -3x - 4$

$y = 3x + 4$

$m = 3$

$\perp m = -\frac{1}{3}$

Perpendicular if $m_1 m_2 = -1$

$y = mx + b$

$-3 = -\frac{1}{3}(-5) + b$

$b = -\frac{14}{3}$

$y = -\frac{1}{3}x - \frac{14}{3}$

$3y = -x - 14$

$\boxed{x + 3y = -14}$

General $Ax + By = C$

Mini-Lecture 1.5

Circles

Learning Objectives:

1. Write the Standard Form of the Equation of a Circle (p. 45)
2. Graph a Circle by Hand and by Using a Graphing Utility. (p. 46)
3. Work with the General Form of the Equation of a Circle (p. 48)

Examples:

1.. Write the standard form of the equation of the circle with center (-8, 5) and radius $\sqrt{11}$.

2. Find the center and the radius of the circle, graph the circle, find any intercepts.

$$(x-2)^2+(y+1)^2=9$$

3. Find the general form of the equation of the circle with center $(0,3)$ and containing the point $(2,-1)$.

4. Write the equation of the circle in standard form and give the center and radius.

$$2x^2+2y^2-8x+12y-6=0$$

Mini Lecture 1.5

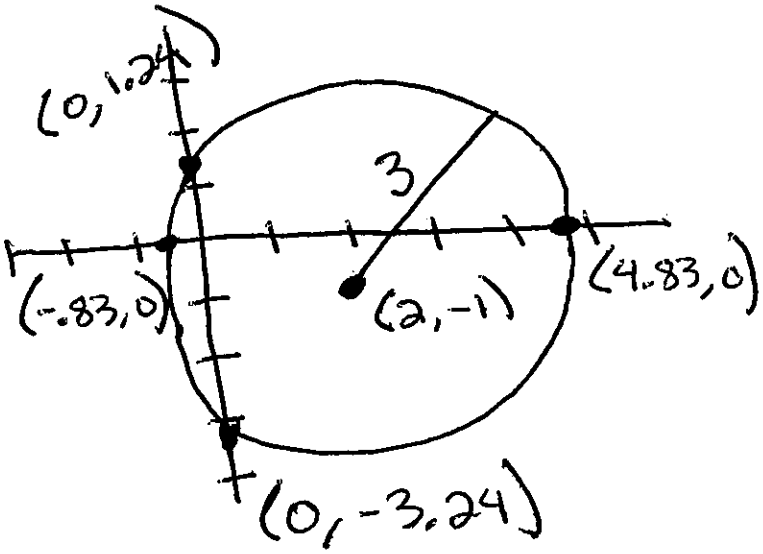
① Standard form circle center $(-8, 5)$ radius $\sqrt{11}$.

$$(x-h)^2 + (y-k)^2 = r^2$$

Center (h, k)

$$\boxed{(x+8)^2 + (y-5)^2 = 11}$$

② Graph $(x-2)^2 + (y+1)^2 = 9$ and find all intercepts



x-int

$(x-2)^2 + ((0)+1)^2 = 9$

$(x-2)^2 = 8$

$x = 2 \pm 2\sqrt{2}$

$x \approx 4.83, -.83$

y-int

$(0-2)^2 + (y+1)^2 = 9$

$(y+1)^2 = 5$

$y = -1 \pm \sqrt{5}$

$y \approx 1.24, -3.24$

Center $(2, -1)$

$r = 3$

(3) General Form of the circle with center $(0,3)$ containing $(2,-1)$

$$d = r = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$r = \sqrt{(2-0)^2+(-1-3)^2}$$

$$r = \sqrt{20} = \boxed{2\sqrt{5} \text{ radius}}$$

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-0)^2+(y-3)^2=20$$

$$x^2+y^2-6y+9=20$$

$$\boxed{x^2+y^2-6y-11=0}$$

General Form

$$x^2+y^2+ax+by+c=0$$

mini lecture
1.5 continued

(4) write in standard form

$2x^2+2y^2-8x+12y-6=0$

$x^2+y^2-4x+6y-3=0$

$x^2-4x+\boxed{4}+y^2+6y+\boxed{9}=3+\boxed{4}+\boxed{9}$

$(x-2)^2+(y+3)^2=16$

center $(2,-3)$

radius $=4$

Mini-Lecture 5.1

Composite Functions

Learning Objectives:

1. Form a Composite Function (p. 251)
2. Find the Domain of a Composite Function (p. 252)

Examples:

1. $f(x) = -x^2 + 3x - 4$ and $g(x) = 2 - x$, find $(f \circ g)(x)$ and determine its domain.

2. $g(x) = \dfrac{3}{x-2}$, find $(g \circ g)(5)$.

3. Find the domain of the composite function $f \circ g$. $f(x) = x + 3$; $g(x) = \sqrt{x+2}$

4. Find functions f and g so that $f \circ g = H$. $H(x) = \sqrt{3x-4}$

Mini Lecture 5.1

① $f(x) = -x^2 + 3x - 4$

$g(x) = 2 - x$

find $(f \circ g)(x)$ and determine domain

$$(f \circ g)(x) = -(2-x)^2 + 3(2-x) - 4$$

$$= -4 + 4x - x^2 + 6 - 3x - 4$$

$$= \boxed{-x^2 + x - 2}$$

$\boxed{\text{domain } (-\infty, \infty)}$

② $g(x) = \dfrac{3}{x-2}$ find $(g \circ g)(5)$

$$g(5) = \frac{3}{(5-2)} = 1$$

$$g(1) = \frac{3}{(1-2)} = \boxed{-3}$$

mini lecture
5.1 continued

(3) Find domain $(f \circ g)(x)$

$f(x) = x+3 \longrightarrow$ domain $(-\infty, \infty)$

$g(x) = \sqrt{x+2} \longrightarrow$ domain $[-2, \infty)$

$(f \circ g)(x) = (\sqrt{x+2}) + 3$

$= \boxed{\sqrt{x+2} + 3}$

$\boxed{\text{Domain } [-2, \infty)}$

(4) Find functions f and g so that $f \circ g = H$

$H(x) = \sqrt{3x-4}$

let $f(x) = \sqrt{x}$ ✓

let $g(x) = 3x-4$

$(f \circ g)(x) = \boxed{\sqrt{(3x-4)}} = H$