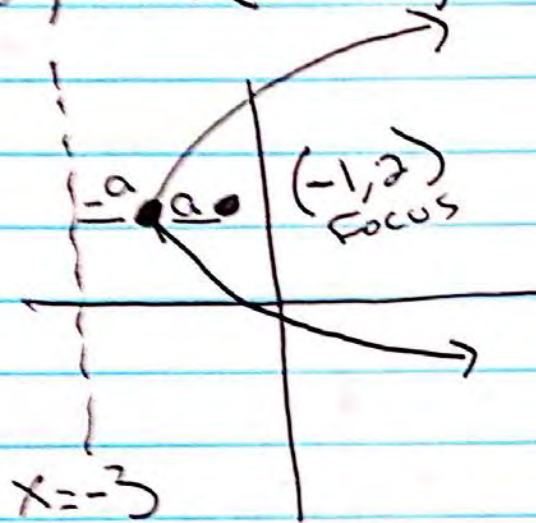


10.2 min: lecture

①  $(y-2)^2 = 4(x+2)$

$y^2 = 4ax$



$4a = 4$   
 $a = 1$

~~Equation~~  $(y-k)^2 = 4a(x-h)$

vertex  $(h, k) = (-2, 2)$

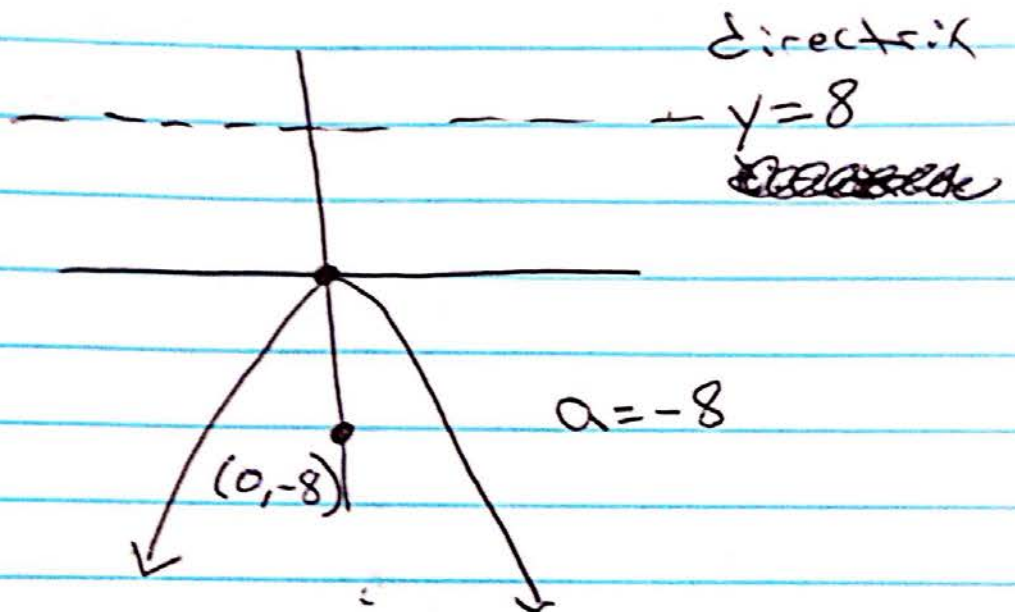
Focus  $(h+a, k) = (-1, 2)$

directrix:  $x = X = h - a \rightarrow x = -3$

## 10.2 mini lecture

② Focus  $(0, -8)$

directrix:  $y = 8$



$$(x-h)^2 = 4a(y-k)$$

$$(x-0)^2 = 4(-8)(y-0)$$

$$x^2 = -32y$$

$$y = -\frac{x^2}{32}$$

## 10.2 mini Lecture

③  $x^2 + 2x + 8y + 1 = 0$

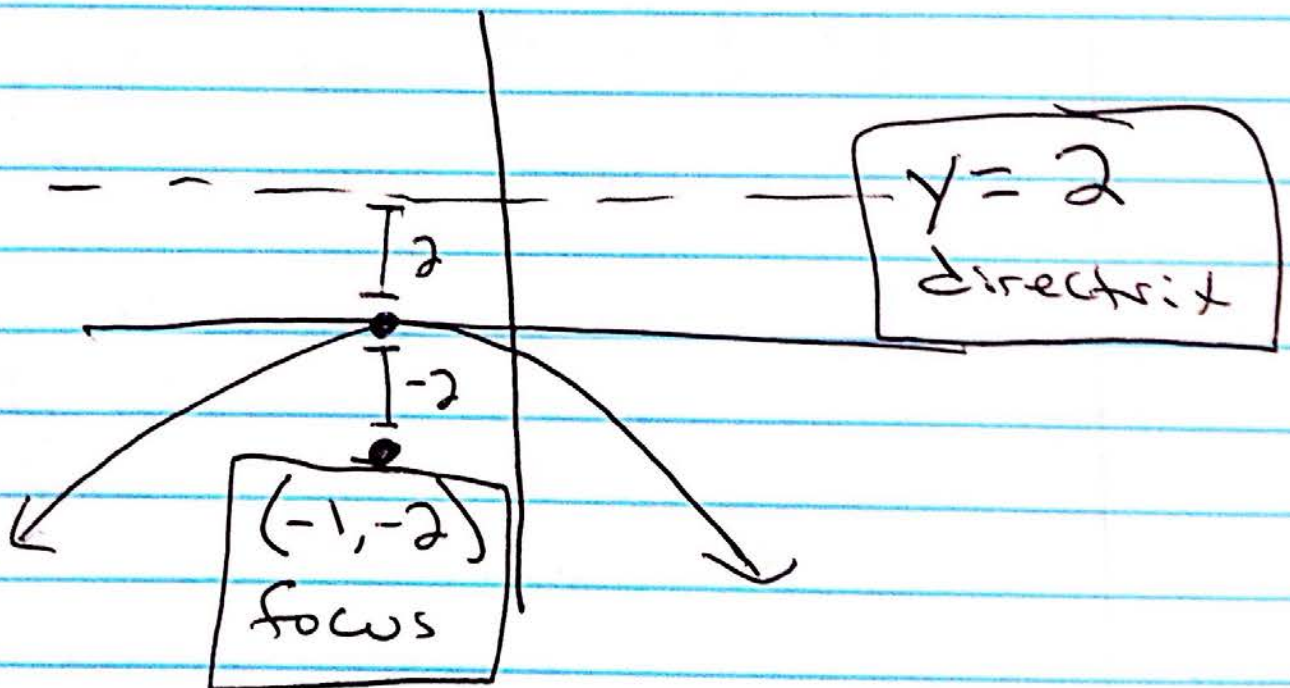
$$x^2 + 2x + 1 = -8y$$

$$(x+1)^2 = -8y$$

$$\text{vertex } (-1, 0)$$

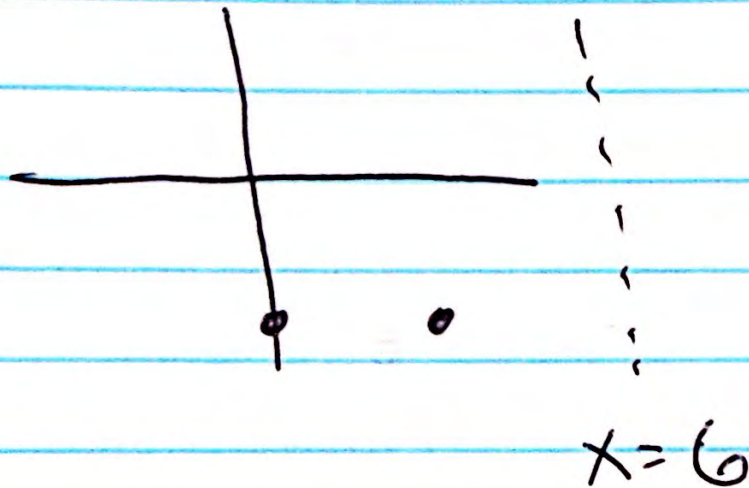
$$4a = -8$$

$$a = -2$$



10.2 min: Lecture

(4) vertex =  $(3, -4)$   
focus  $(0, -4)$



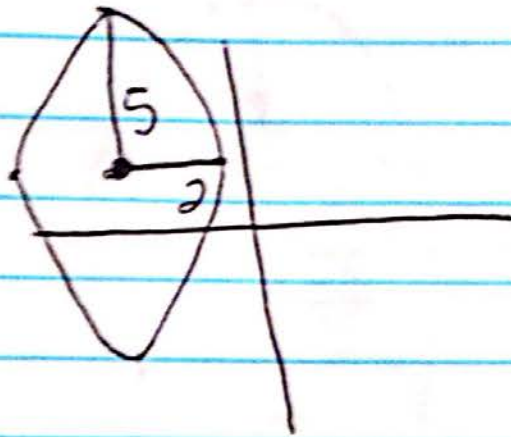
$$(y+4)^2 = 4(-3)(x-3)$$

$$(y+4)^2 = -12(x-3)$$

### 10.3 mini lecture

$$\textcircled{1} \quad 25(x+3)^2 + 4(y-2)^2 = 100$$

$$\frac{(x+3)^2}{4} + \frac{(y-2)^2}{25} = 1$$



major = 10

minor = 4

center  $(-3, 2)$

$$\textcircled{2} \quad 9x^2 + 4y^2 - 36x + 32y + 64 = 0$$

$$9x^2 - 36x + 4y^2 + 32y = -64$$

$$9(x^2 - 4x + 4) + 4(y^2 + 8y + 16) = +36 + 64$$

$$9(x-2)^2 + 4(y+4)^2 = 36$$

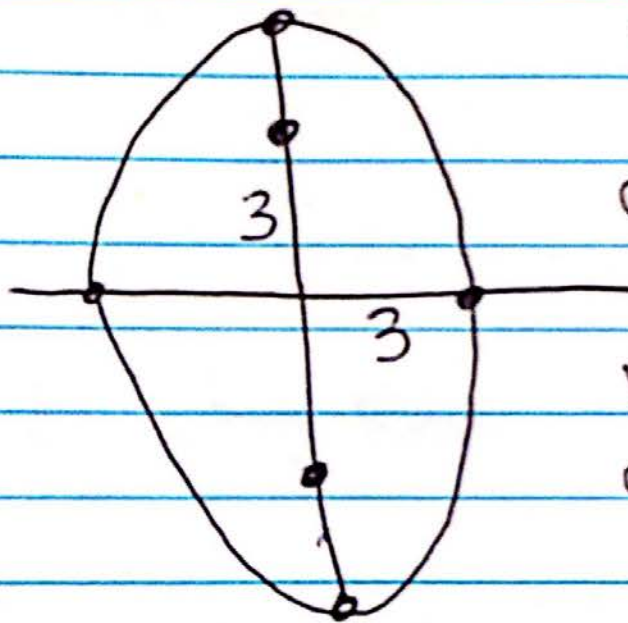
$$\frac{(x-2)^2}{4} + \frac{(y+4)^2}{9} = 1$$

center  $(2, -4)$  major = 6

### 10.3 mini lecture

③ foci:  $(0, 3)$ ,  $(0, -3)$

x-intercepts  $(-3 \text{ and } 3)$



$$b^2 = a^2 - c^2$$

$$9 = a^2 - 9$$

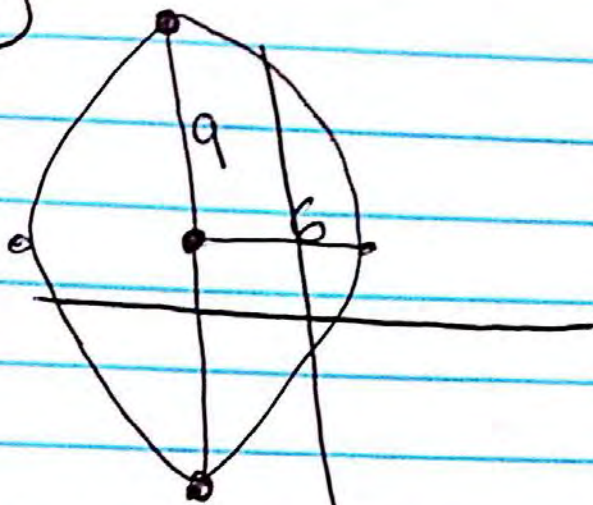
$$18 = a^2$$

$$a = \pm 3\sqrt{2}$$

$$\frac{x^2}{9} + \frac{y^2}{18} = 1$$

### 10.3 mini lecture

4



major = 18 vertical  
minor = 12 horizontal

center  $(-1, 2)$

$$\frac{(x+1)^2}{36} + \frac{(y-2)^2}{81} = 1$$

$$2a = 18$$

$$a = 9$$

$$a^2 = 81$$

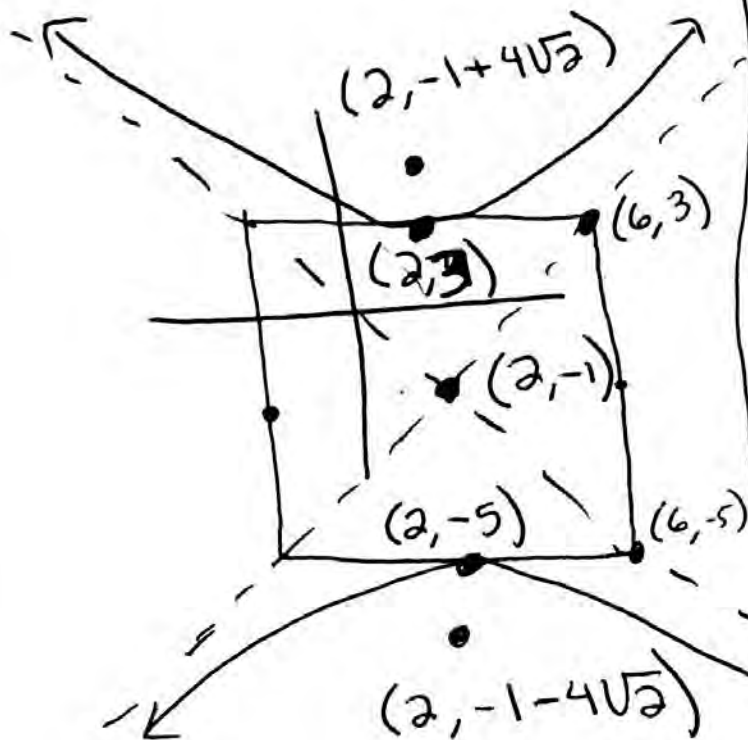
$$2b = 12$$

$$b = 6$$

$$b^2 = 36$$

10.4 min: lecture

$$\textcircled{1} \frac{(y+1)^2}{16} - \frac{(x-2)^2}{16} = 1$$



$$a = 4 \quad c^2 = a^2 + b^2$$
$$b = 4$$
$$c = \pm 4\sqrt{2}$$

center  $(2, -1)$

vertex  $(2, 3), (2, -5)$

foci  $(2, -1 - 4\sqrt{2})$   
 $(2, -1 + 4\sqrt{2})$

asymptote:

~~$y = mx + b$~~   $y = mx + b$

$$y = x - 3$$

$$y = -x + 1$$



10.4 min: lecture

②  $25x^2 - 4y^2 - 32y - 164 = 0$

$25x^2 - 4(y^2 + 8y)$

$25x^2 - 4(y^2 + 8y) = 164$

$25x^2 - 4(y^2 + 8y + 16) = 164 + (-64)$

$25x^2 - 4(y+4)^2 = 100$

$\frac{x^2}{4} - \frac{(y+4)^2}{25} = 1$

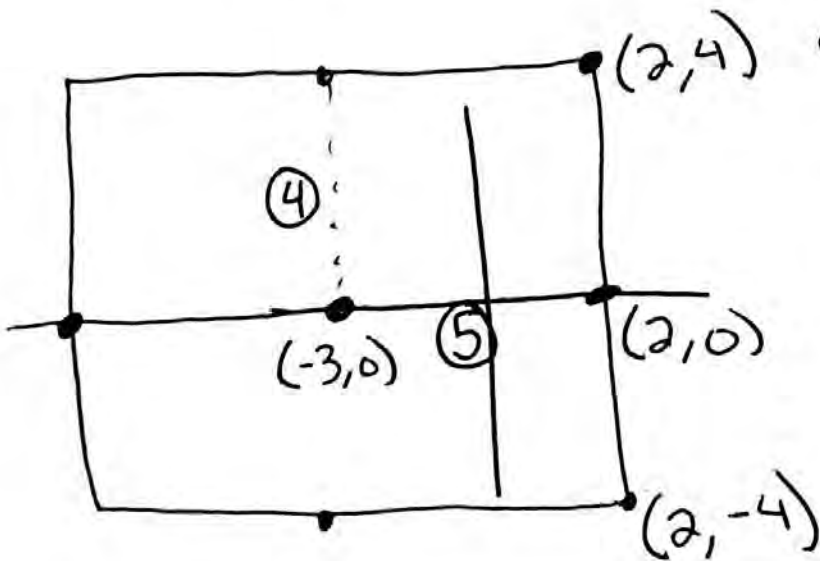
Center  
(0, -4)

③  $\frac{(x+3)^2}{25} - \frac{y^2}{16} = 1$

$a=5$

$b=4$

center (-3, 0)



$y = mx + b$

$4 = \frac{4}{5}(2) + b$

$b =$

$y = \frac{4}{5}x + \frac{12}{5}$

$y = mx + b$

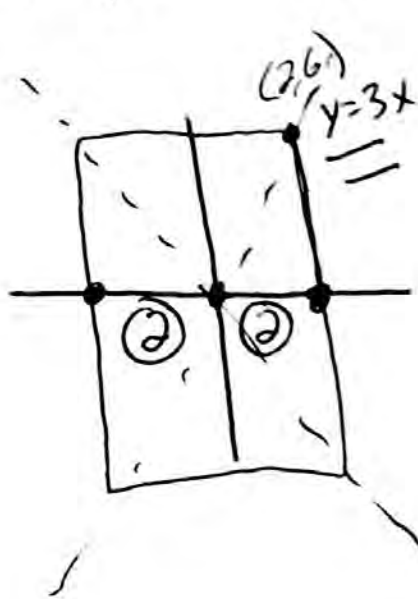
$-4 = -\frac{4}{5}(2) + b$

$y = -\frac{4}{5}x - \frac{12}{5}$

asymptotes



④ endpoints transverse axis  $(-2, 0)$   
 $(2, 0)$   
 asymptote  $y = 3x$



center =  $(0, 0)$

$$\frac{x^2}{4} - \frac{y^2}{36} = 1$$

$a = 4$ , ~~width~~  
 $b = 6$

10.5 min: lecture

①  $x^2 - 12y - 4x + 30 = 0$

parabola

only has an  $x^2$ ,  
no  $y^2$

10.5 mini lecture

②  $x^2 - 6xy + y^2 - 8 = 0$      $\theta = 45^\circ$

$$x = x' \cos 45 - y' \sin 45$$

$$y = x' \sin 45 + y' \cos 45$$

$$x = x' \frac{\sqrt{2}}{2} - y' \frac{\sqrt{2}}{2}$$

$$y = x' \frac{\sqrt{2}}{2} + y' \frac{\sqrt{2}}{2}$$

$$\left(x' \frac{\sqrt{2}}{2} - y' \frac{\sqrt{2}}{2}\right)^2 - 6\left(x' \frac{\sqrt{2}}{2} - y' \frac{\sqrt{2}}{2}\right)\left(x' \frac{\sqrt{2}}{2} + y' \frac{\sqrt{2}}{2}\right) + \left(x' \frac{\sqrt{2}}{2} + y' \frac{\sqrt{2}}{2}\right)^2 - 8 = 0$$

---

$$\begin{aligned} \left(x' \frac{\sqrt{2}}{2} - y' \frac{\sqrt{2}}{2}\right)\left(x' \frac{\sqrt{2}}{2} - y' \frac{\sqrt{2}}{2}\right) &= \frac{1}{2}x'^2 - x'y' + \frac{1}{2}y'^2 \\ -6\left(x' \frac{\sqrt{2}}{2} - y' \frac{\sqrt{2}}{2}\right)\left(x' \frac{\sqrt{2}}{2} + y' \frac{\sqrt{2}}{2}\right) &= -3x'^2 + 3y'^2 \\ \left(x' \frac{\sqrt{2}}{2} + y' \frac{\sqrt{2}}{2}\right)\left(x' \frac{\sqrt{2}}{2} + y' \frac{\sqrt{2}}{2}\right) &= \frac{1}{2}x'^2 + x'y' + \frac{1}{2}y'^2 - 8 \end{aligned}$$

---

$$4y'^2 - 2x'^2 = 8$$

$$\frac{y'^2}{2} - \frac{x'^2}{4} = 1$$

10.5 mini lecture

(3)  $12x^2 - 6\sqrt{3}xy + 18y^2 - 63 = 0$

~~AB~~  
 $X = X' \cos \frac{\pi}{6} - Y' \sin \frac{\pi}{6}$

$Y = X' \sin \frac{\pi}{6} + Y' \cos \frac{\pi}{6}$

$$x = \frac{X'\sqrt{3}}{2} - \frac{Y'}{2}$$

$$y = \frac{X'}{2} + \frac{Y'\sqrt{3}}{2}$$

$$\cot 2\theta = \frac{A-C}{B}$$

$$\cot 2\theta = \frac{12-18}{-6\sqrt{3}}$$

$$\cot 2\theta = \frac{-6}{-6\sqrt{3}}$$

$$\cot 2\theta = \frac{1}{\sqrt{3}}$$

$$\cot 2\theta = \frac{\sqrt{3}}{3}$$

$$2\theta = \frac{\pi}{3}$$

$$\theta = \frac{\pi}{6}$$

$$12 \left( \frac{\sqrt{3}}{2} X' - \frac{1}{2} Y' \right) \left( \frac{\sqrt{3}}{2} X' - \frac{1}{2} Y' \right) = 9X'^2 - 6\sqrt{3}X'Y' + 3Y'^2$$

$$-6\sqrt{3} \left( \frac{X'\sqrt{3}}{2} - \frac{Y'}{2} \right) \left( \frac{X'}{2} + \frac{Y'\sqrt{3}}{2} \right) = -\frac{9}{2}X'^2 + \frac{9}{2}Y'^2 - 3\sqrt{3}X'Y'$$

$$18 \left( \frac{X'}{2} + \frac{Y'\sqrt{3}}{2} \right) \left( \frac{X'}{2} + \frac{Y'\sqrt{3}}{2} \right) = \frac{9}{2}X'^2 + 9\sqrt{3}X'Y' + \frac{27}{2}Y'^2$$

$$9X'^2 + 21Y'^2 - 63 = 0$$

$$9X'^2 + 21Y'^2 = 63$$

$$\frac{X'^2}{7} + \frac{Y'^2}{3} = 1$$

10.6 min: Lecture

$$\textcircled{1} \quad r = \frac{4}{3-6\sin\theta} \rightarrow r = \frac{\frac{4}{3}}{1-2\sin\theta}$$



$$r = \frac{ep}{1-e\sin\theta}$$

$$e = 2$$

hyperbola

$$e > 1$$

$$\textcircled{2} \quad r = \frac{1}{1+2\cos\theta}$$

$$r = \frac{ep}{1+e\cos\theta}$$

$$e = 2$$

hyperbola

$$e > 1$$

\*note  $e = 1$  parabola

$e > 1$  hyperbola

$e < 1$  ~~ellipse~~  
ellipse

10.6 mini lecture

③

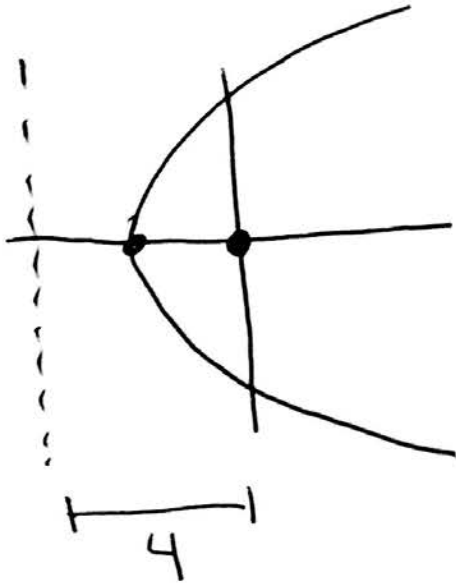
$$r = \frac{4}{1 - \cos \theta}$$

$$e = 1$$

$$ep = 4$$

$$p = 4$$

$e = 1$  parabola



④

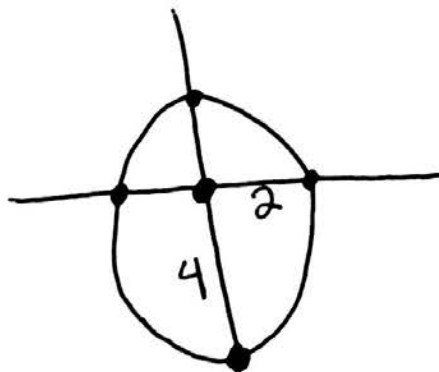
$$r = \frac{8}{4 + 2 \sin \theta}$$

$$e = \frac{1}{2}$$

$$ep = 2 \quad p = 4$$

$$r = \frac{2}{1 + \frac{1}{2} \sin \theta}$$

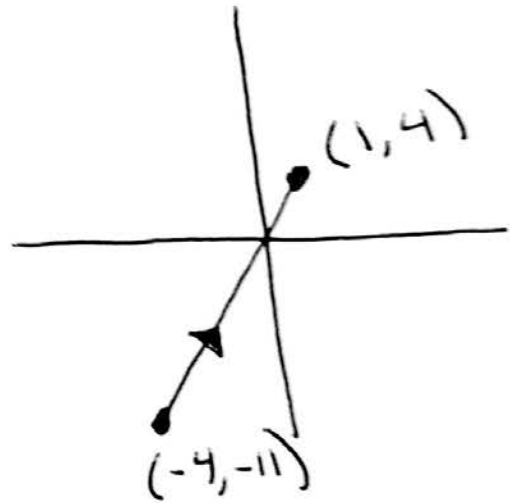
$e < 1$  ellipse



10.7 min: Lecture

①  $x = t - 1$ ,  $y = 3t - 2$ ,  $-3 \leq t \leq 2$

$t$	$x$	$y$
-3	-4	-11
-2	-3	-8
-1	-2	-5
0	-1	-2
1	0	1
2	1	4
<del>3</del>		



②  $x = -2 + \cos t$ ,  $y = 2 + \sin t$   $0 \leq t < 2\pi$

$$(x+2) = \cos t$$

$$(y-2) = \sin t$$

$$(x+2)^2 = (\cos t)^2$$

$$(y-2)^2 = (\sin t)^2$$

$$(x+2)^2 + (y-2)^2 = 1$$

$$\sin^2 t + \cos^2 t = 1$$



10.7 mini lecture

③  $(6, -4), (-2, 1)$

$$x = (-2 - 6)t + 6$$

$$x = -8t + 6$$

$$y = (1 - (-4))t - 4$$

$$y = 5t - 4$$

OR

$$x = (6 - (-2))t - 2$$

$$x = 8t - 2$$

$$y = (-4 - 1)t + 1$$

$$y = -5t + 1$$

④  $x = t^2 + 2t + 3$   
 $y = -t$

(use calculator)

