Are you Ready for Calculus

1) a) $$\frac{x^3-9x}{x^2-7x+12} = \frac{x(x^2-9)}{(x-4)(x-3)} = \frac{x\cancel{(x-3)}(x+3)}{(x-4)\cancel{(x-3)}} = \boxed{\frac{x^2+3x}{x-4}}$$

b) $$\frac{x^2-2x-8}{x^3+x^2-2x} = \frac{(x-4)(x+2)}{x(x^2+x-2)} = \frac{(x-4)\cancel{(x+2)}}{x\cancel{(x+2)}(x-1)} = \boxed{\frac{x-4}{x^2-x}}$$

c) $$\frac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}} = \frac{\frac{5-x}{5x}}{\frac{25-x^2}{25x^2}} = \frac{\frac{5-x}{5x}}{\frac{(5-x)(5+x)}{25x^2}}$$

$$= \frac{\cancel{5-x}}{5x} \cdot \frac{25x^2}{\cancel{(5-x)}(5+x)}$$ reduce

$$= \boxed{\frac{5x}{x+5}}$$

d) $$\frac{9-x^{-2}}{3+x^{-1}} = \frac{9-\frac{1}{x^2}}{3+\frac{1}{x}} = \frac{\frac{9x^2-1}{x^2}}{\frac{3x+1}{x}}$$

$$= \frac{\frac{(3x-1)(3x+1)}{x^2}}{\frac{3x+1}{x}}$$

$$= \frac{\cancel{(3x+1)}(3x-1)}{x^2} \cdot \frac{x}{\cancel{(3x+1)}}$$ reduce

$$= \boxed{\frac{3x-1}{x}}$$

2) a) $\frac{2}{\sqrt{3}+\sqrt{2}} = \frac{2(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{2(\sqrt{3}-\sqrt{2})}{1} = \boxed{2(\sqrt{3}-\sqrt{2})}$

b) $\frac{4}{1-\sqrt{5}} = \frac{4(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})} = \frac{4(1+\sqrt{5})}{-4} = \boxed{-1-\sqrt{5}}$

c) $\frac{1}{1+\sqrt{3}-\sqrt{5}} = \frac{1((1+\sqrt{3})+\sqrt{5})}{((1+\sqrt{3})-\sqrt{5})((1+\sqrt{3})+\sqrt{5})}$

$$= \frac{1+\sqrt{3}+\sqrt{5}}{1+2\sqrt{3}+3-5} = \frac{1+\sqrt{3}+\sqrt{5}}{-1+2\sqrt{3}}$$

$$= \frac{(1+\sqrt{3}+\sqrt{5})(-1-2\sqrt{3})}{(-1+2\sqrt{3})(-1-2\sqrt{3})}$$

$$= \boxed{\frac{7+3\sqrt{3}+\sqrt{5}+2\sqrt{15}}{11}}$$

3 a) $\dfrac{(2a^2)^3}{b} = \dfrac{2^3 a^6}{b} = \boxed{8a^6 b^{-1}}$

b) $\sqrt{9ab^3} = \sqrt{9}\, a^{1/2} b^{3/2} = \boxed{3a^{1/2} b^{3/2}}$

c) $\dfrac{a(2/b)}{3/a} = \dfrac{2a}{b} \cdot \dfrac{a}{3} = \boxed{\dfrac{2}{3} a^2 b^{-1}}$

d) $\dfrac{ab-a}{b^2-b} = \dfrac{a(b-1)}{b(b-1)} = \boxed{ab^{-1}}$

e) $\dfrac{a^{-1}}{b^{-1}\sqrt{a}} = \dfrac{b}{a a^{1/2}} = \dfrac{b}{a^{3/2}} = \boxed{a^{-3/2} b}$

f) $\left(\dfrac{a^{2/3}}{b^{1/2}}\right)^2 \left(\dfrac{b^{3/2}}{a^{1/2}}\right) = \dfrac{a^{4/3}}{b} \cdot \dfrac{b^{3/2}}{a^{1/2}} = \boxed{a^{5/6} b^{1/2}}$

4) a) $5^{x+1} = 25$ $\quad 5^{x+1} = 5^2$ $\quad x+1=2$ $\quad \boxed{x = 1}$

b) $\frac{1}{3} = 3^{2x+2}$ $\quad 3^{-1} = 3^{2x+2}$ $\quad -1 = 2x+2$ $\quad \boxed{x = -\frac{3}{2}}$

c) $\log_2 x = 3$ $\quad 2^3 = x$ $\quad \boxed{x = 8}$

d) $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

$\log_3 x^2 = \log_3 4^2 - \log_3 5^4$

$\log_3 x^2 = \log_3 \frac{16}{625}$

$x^2 = \frac{16}{625}$ $\quad \boxed{x = \pm \frac{4}{25}}$

5 a) $\log_2 5 + \log_2 (x^2-1) - \log_2 (x-1)$

$$\log_2 \frac{5(x^2-1)}{x-1} = \log_2 \frac{5(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$= \boxed{\log_2 5(x+1)}$$

b) $2\log_4 9 - \log_2 3$

* note $\log_4 9 = \log_2 3$

$$2\log_2 3 - \log_2 3 = \boxed{\log_2 3}$$

c) $3^{(2\log_3 5)} = 3^{\log_3 25} = \boxed{25}$

6

a) $\log_{10} 10^{1/2} = \boxed{\frac{1}{2}}$

b) $\log_{10} \frac{1}{10^x} = \log_{10} 10^{-x} = \boxed{-x}$

c) $2\log_{10}\sqrt{x} + 3\log_{10} x^{1/3}$

$\log_{10}(\sqrt{x})^2 + \log_{10}(x^{1/3})^3$

$\log_{10} x + \log_{10} x = \boxed{2\log_{10} x}$

⑦ a) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, for a

$$\frac{x}{a} = 1 - \frac{y}{b} - \frac{z}{c}$$

$$x = \left(1 - \frac{y}{b} - \frac{z}{c}\right)a$$

$$\frac{x}{1 - \frac{y}{b} - \frac{z}{c}} = a$$

$$\frac{x}{\frac{BC - cy - bz}{bc}} = \boxed{\frac{bcx}{BC - cy - BZ}}$$

b) $2(ab + bc + ca) = V$, for a

$$ab + bc + ca = \frac{V}{2}$$

$$ab + ac = \frac{V}{2} - bc$$

$$a(b + c) = \frac{V}{2} - bc$$

$$a = \frac{V}{2(b+c)} - \frac{bc}{b+c}$$

$$\boxed{a = \frac{V - 2bc}{2(b+c)}}$$

7c) $A = 2\pi r^2 + 2\pi r h$, for positive r

$0 = 2\pi r^2 + 2\pi r h - A$

* let $r = x$

use quadratic formula

$$r = \frac{-2\pi h \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-A)}}{2(2\pi)}$$

$$r = \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 + 8\pi A}}{4\pi}$$

$$r = \frac{-2\pi h \pm \sqrt{4(\pi^2 h^2 + 2\pi A)}}{4\pi}$$

$$r = \frac{-2\pi h \pm 2\sqrt{\pi^2 h^2 + 2\pi A}}{4\pi}$$

$$\boxed{r = \frac{-\pi h + \sqrt{\pi^2 h^2 + 2\pi A}}{2\pi}}$$

7 d) $A = P + nrP$, For P

$A = P(1 + nr)$

$\boxed{P = \frac{A}{1+nr}}$

e) $2x - 2yd = y + xd$, for d

$2x - y = xd + 2yd$

$2x - y = d(x + 2y)$

$\boxed{\frac{2x-y}{x+2y} = d}$

7 f) $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$, for x

$$-\frac{2x}{4\pi} = \frac{1-x}{2}$$

$$-4x = 4\pi - 4\pi x$$

$$-4x + 4\pi x = 4\pi$$

$$-x + \pi x = \pi$$

$$x(\pi - 1) = \pi$$

$$\boxed{x = \frac{\pi}{\pi - 1}}$$

8 a) $y = x^2 + 4x + 3$

$y = x^2 + 4x + \boxed{4} + 3 - \boxed{4}$

$y = (x+2)^2 - 1$

$\boxed{y - (-1) = (x - (-2))^2}$

b) $3x^2 + 3x + 2y = 0$

$3x^2 + 3x = -2y$

$3\left(x^2 + x + \boxed{1/4}\right) = -2y + \boxed{3/4}$

$3\left(x + \frac{1}{2}\right)^2 = -2y + \frac{3}{4}$

$-\frac{3}{2}\left(x + \frac{1}{2}\right)^2 = y - \frac{3}{8}$

$\boxed{-\frac{3}{2}\left(x - \left(-1/2\right)\right)^2 = y - \frac{3}{8}}$

8 c) $9y^2-6y-9-x=0$

$9y^2-6y \quad = x+9$

$y^2-\frac{2}{3}y+\boxed{\frac{1}{9}} = \frac{x}{9}+1+\boxed{1/9}$

$\left(y-\frac{1}{3}\right)^2 = \frac{x}{9}+\frac{10}{9}$

$\boxed{9\left(y-\frac{1}{3}\right)^2 = x-(-10)}$

9 a) x^6-16x^4

$x^4(x^2-16)$

$\boxed{x^4(x+4)(x-4)}$

9) b) $4x^3-8x^2-25x+50$

$4x^2(x-2)-25(x-2)$

$(4x^2-25)(x-2)$

$\boxed{(2x+5)(2x-5)(x-2)}$

c) $8x^3+27$

$\boxed{(2x+3)(4x^2-6x+9)}$

d) x^4-1

$(x^2-1)(x^2+1)$

$\boxed{(x+1)(x-1)(x^2+1)}$

10 a) $x^6 - 16x^4 = 0$

$x^4(x^2 - 16) = 0$

$x^4(x+4)(x-4) = 0$

$\boxed{x = 0, 4, -4}$

b) $4x^3 - 8x^2 - 25x + 50 = 0$

$4x^2(x-2) - 25(x-2) = 0$

$(4x^2 - 25)(x-2) = 0$

$(2x+5)(2x-5)(x-2) = 0$

$\boxed{x = 2, -5/2, 5/2}$

c) $8x^3 + 27 = 0$

$(2x+3)(4x^2 - 6x + 9)$

$\boxed{x = -3/2}$

11 a) $3\sin^2 x = \cos^2 x \qquad 0 \le x < 2\pi$

$3\sin^2 x = 1 - \sin^2 x$

$4\sin^2 x - 1 = 0$

$(2\sin x - 1)(2\sin x + 1) = 0$

$\sin x = 1/2 \qquad \sin x = -1/2$

$$\boxed{x = \pi/6,\ \frac{5\pi}{6},\ \frac{7\pi}{6},\ \frac{11\pi}{6}}$$

b) $\cos^2 x - \sin^2 x = \sin x \qquad -\pi < x \le \pi$

$(1 - \sin^2 x) - \sin^2 x = \sin x$

$1 - 2\sin^2 x = \sin x$

$0 = 2\sin^2 x + \sin x - 1$

$0 = (2\sin x - 1)(\sin x + 1)$

$\sin x = 1/2 \qquad \sin x = -1$

$$\boxed{x = \frac{-\pi}{2},\ \pi/6,\ 5\pi/6}$$

(11) c) $\tan x + \sec x = 2\cos x$ $\quad (-\infty, \infty)$

$$\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$$

$$\sin x + 1 = 2\cos^2 x$$

$$\sin x + 1 = 2(1 - \sin^2 x)$$

$$\sin x + 1 = 2 - 2\sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$\sin x = 1/2 \qquad \sin x = -1$

$$\boxed{\begin{array}{l} x = \pi/6 + 2\pi k \\[1em] x = 5\pi/6 + 2\pi k \end{array}}$$

* $\tan \frac{3\pi}{2}$ undefined

12 a) $\cos 210^\circ = \boxed{-\frac{\sqrt{3}}{2}}$

b) $\sin 5\pi/4 = \boxed{-\frac{\sqrt{2}}{2}}$

c) $\tan^{-1}(-1) = \boxed{-\pi/4}$

d) $\sin^{-1}(-1) = \boxed{-\pi/2}$

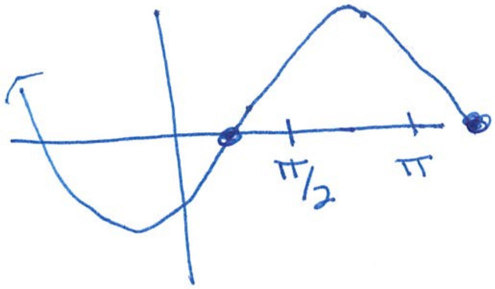
e) $\cos 9\pi/4 = \boxed{\frac{\sqrt{2}}{2}}$

f) $\sin^{-1} \frac{\sqrt{3}}{2} = \boxed{\pi/3}$

g) $\tan 7\pi/6 = \boxed{\frac{\sqrt{3}}{3}}$

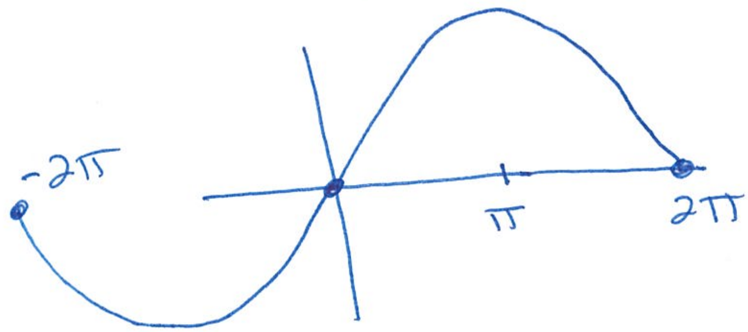
h) $\cos^{-1}(-1) = \boxed{\pi}$

13 a) $\sin\left(x - \frac{\pi}{4}\right)$



Shift right $\pi/4$

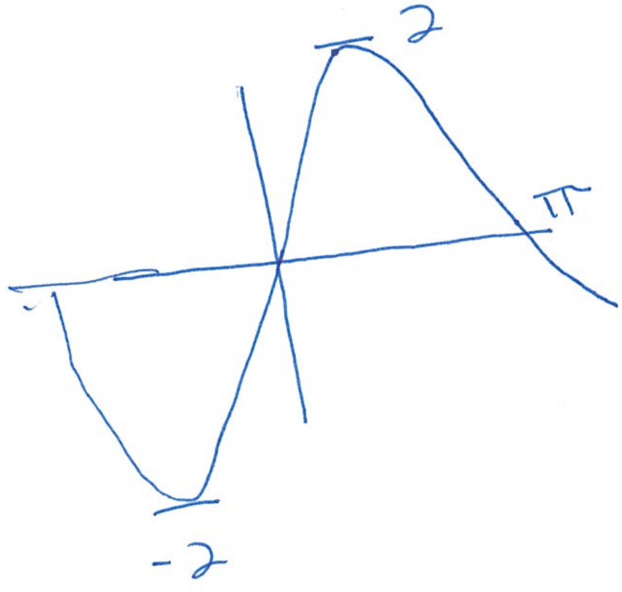
b) $\sin \frac{x}{2}$



*

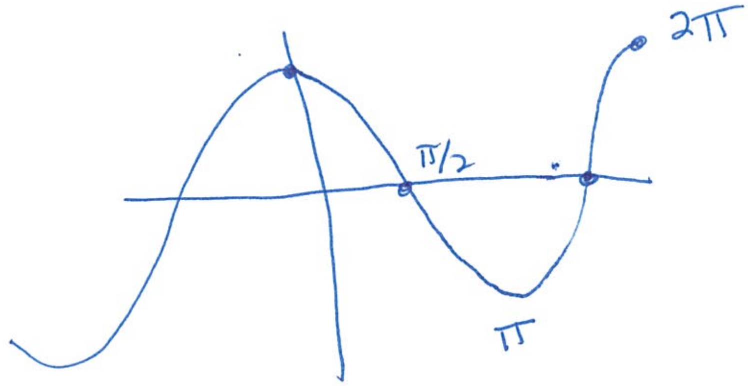
widen by 2

c) $2 \sin x$



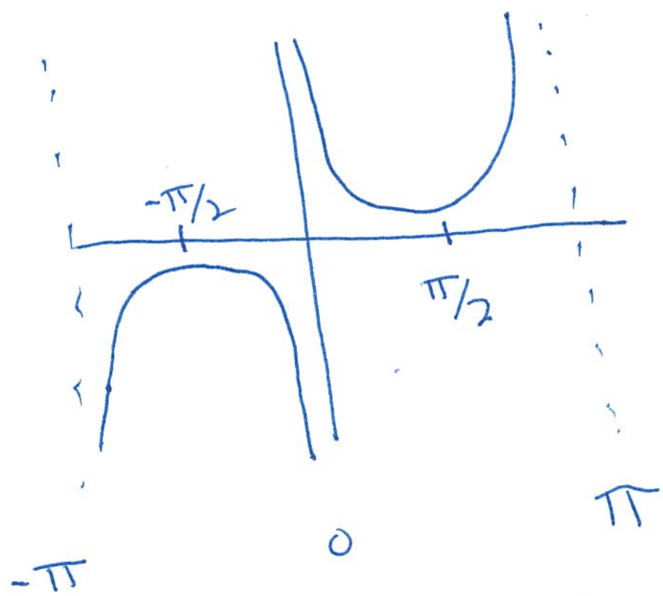
stretch by 2

(13) d) cos x



e) $\frac{1}{\sin x}$

$\underline{\underline{\csc x}}$



14 a) $4x^2+12x+3=0$

$$x=\frac{-12\pm\sqrt{(12)^2-4(4)(3)}}{2(4)}$$

$$x=\frac{-12\pm\sqrt{96}}{8}=\frac{-12\pm4\sqrt{6}}{8}$$

$$=\boxed{\frac{-3\pm\sqrt{6}}{2}}$$

b) $2x+1=\frac{5}{x+2}$

$(2x+1)(x+2)=5$

$2x^2+5x+2=5$

$2x^2+5x-3=0$

$(2x-1)(x+3)=0$

$\boxed{x=1/2,\ x=-3}$

(14) c) $\frac{x+1}{x} - \frac{x}{x+1} = 0$

$(x+1)(x+1) - x(x) = 0$

$x^2 + 2x + 1 - x^2 = 0$

$2x + 1 = 0$

$\boxed{x = -1/2}$

(15) a) $x^5 - 4x^4 + x^3 - 7x + 1$, by $x+2$

$$\begin{array}{r|rrrrrr} -2 & 1 & -4 & 1 & 0 & -7 & 1 \\ & \downarrow & -2 & 12 & -26 & 52 & -90 \\ \hline & 1 & -6 & 13 & -26 & 45 & -89 \end{array}$$

$\boxed{-89 \text{ remainder}}$

15 b)

$$
\begin{array}{r}
x^2 - x + 1 \\
x^3 + 0x^2 + 0x + 1 \;\big)\; \cancel{x^5} - x^4 + x^3 + 2x^2 - x + 4 \\
-\cancel{x^5} + 0 + 0 - x^2 \\
\hline
-\cancel{x^4} + x^3 + x^2 - \cancel{x} \\
+\cancel{x^4} + 0 + 0 + \cancel{x} \\
\hline
\cancel{x^3} + x^2 + 0x + 4 \\
-\cancel{x^3} + 0 + 0 - 1 \\
\hline
x^2 + 3
\end{array}
$$

$\boxed{x^2 + 3}$ remainder

16 a) $12x^3 - 23x^2 - 3x + 2 = 0$

solution at $x = 2$

$$\begin{array}{r|rrrr} 2 & 12 & -23 & -3 & 2 \\ & \downarrow & 24 & 2 & -2 \\ \hline & 12 & 1 & -1 & 0 \end{array}$$

$12x^2 + x - 1$

$(4x-1)(3x+1) = 0$

$\boxed{x = 1/4, -1/3}$ $\underline{\underline{x = 2}}$

b) $12x^3 + 8x^2 - x - 1 = 0$

zeros

$\dfrac{\pm 1}{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}$

$$\begin{array}{r|rrrr} \frac{1}{3} & 12 & 8 & -1 & -1 \\ & \downarrow & 4 & 4 & 1 \\ \hline & 12 & 12 & 3 & 0 \end{array}$$

$12x^2 + 12x + 3 = 0$

$(6x+3)(2x+1) = 0$

$\boxed{x = -1/2 \quad x = 1/3}$

(17) a) $x^2 + 2x - 3 \le 0$

$(x+3)(x-1) \le 0$

Critical Points at -3 and 1

Between -3 and 1 equation is ≤ 0

$$\boxed{-3 \le x \le 1}$$

(17) b) $\dfrac{2x-1}{3x-2} \leq 1$

$2x-1 \leq 3x-2$

$-x+1 \leq 0$

Critical points
at 1, $2/3$ (from original)

True False True

$2/3$ 1

$$\boxed{x < 2/3 \text{ OR } x \geq 1}$$

17 c) $x^2 + x + 1 > 0$

$\boxed{\text{all real \#'s}}$ makes true

18 a) $|-x+4| \leq 1$

$-x+4 \leq 1$ and $-x+4 \geq -1$

$-x \leq -3$ $\quad$ $-x \geq -5$

$x \geq 3$ $\quad$ $x \leq 5$

$$\boxed{3 \leq x \leq 5}$$

(18) b) $|5x-2| = 8$

$5x-2=8$ $\quad$ $5x-2=-8$

$5x=10$ $\quad$ $5x=-6$

$x=2$ $\quad$ $x=-6/5$

$x = \boxed{2 \text{ OR } -6/5}$

c) $|2x+1| = x+3$

$2x+1=x+3$ and $2x+1=-x-3$

$x=2$ $\quad$ $3x=-4$

$\boxed{x = 2 \text{ OR } -4/3}$

19 a) $(-1, 3)$ and $(2, -4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-3}{2-(-1)} = \frac{-7}{3}$$

$$y = mx + b$$

$$-4 = \frac{-7}{3}(2) + b$$

$$b = \frac{2}{3}$$

$$y = \frac{-7}{3}x + \frac{2}{3}$$

$$3y = -7x + 2$$

$$\boxed{7x + 3y = 2}$$

b) $(-1, 2)$ $\perp$ to $2x - 3y + 5 = 0$

$$-3y = -2x - 5$$

$$y = \frac{2}{3}x + \frac{5}{3}$$

$$m = 2/3$$

$$\perp m = -3/2$$

$$y = -\frac{3}{2}x + b$$

$$2 = -\frac{3}{2}(-1) + b$$

$$2 - \frac{3}{2} = b$$

$$b = \frac{1}{2}$$

$$y = \frac{-3}{2}x + \frac{1}{2}$$

$$\boxed{3x + 2y = 1}$$

19 c) $(2,3)$ midpoint of $(-1,4)$ to $(3,2)$

$$m = \frac{3-3}{1-2} = 0$$

$$\left(\frac{-1+3}{2}, \frac{4+2}{2}\right)$$

$$(1,3)$$

$y = mx + b$

$3 = 2(0) + b$

$b = 3$

$$\boxed{y=3}$$

20 a) $3x - y - 7 = 0$

$-3(x + 5y + 3 = 0)$

$$\begin{array}{r} 3\cancel{x} - y - 7 = 0 \\ -3\cancel{x} - 15y - 9 = 0 \\ \hline -16y = 16 \end{array}$$

$y = -1$

substitute

$3x - (-1) - 7 = 0$

$3x - 6 = 0$

$x = 2$

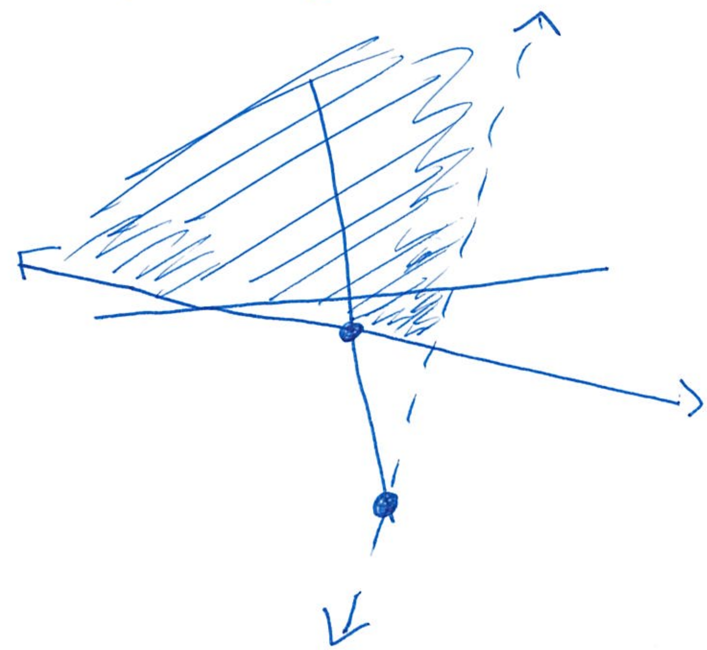
$$\boxed{(2,-1)}$$

20 b) $3x - y - 7 < 0$

$x + 5y + 3 \geq 0$

$y > \frac{1}{3}x - 7$

$y \geq -\frac{1}{5}x - \frac{3}{5}$



21) a) circle $(1,2)$ passes through $(-2,-1)$

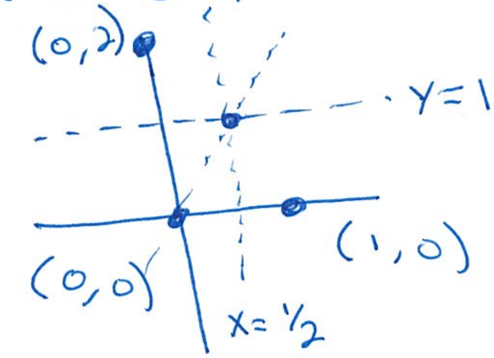
$$r = \sqrt{(-2-1)^2 + (-1-2)^2} = \sqrt{18}$$

$$r^2 = 18$$

$$\boxed{(x-1)^2 + (y-2)^2 = 18}$$

b) passes through origin and x-int at 1

y-int at 2



* take midpoint and draw line $\perp$

intersect at $(1/2, 1)$

* distance between $(1/2, 1)$ and $(0,0)$

$$d = \sqrt{5/4} \rightarrow r = 5/4$$

$$\boxed{\left(x-\frac{1}{2}\right)^2 + (y-1)^2 = \frac{5}{4}}$$

22 $x^2+y^2+6x-4y+3=0$

a) $x^2+6x+\boxed{9}+y^2-4y+\boxed{4}=-3+\boxed{9}+\boxed{4}$

$(x+3)^2+(y-2)^2=10$

$\boxed{\begin{array}{l}\text{center } (-3,2) \\ \text{radius } = \sqrt{10}\end{array}}$

b) tangent at $(-2,5)$

$\perp$ slope $\frac{5-2}{-2-(-3)}=\frac{3}{1}=3 \longrightarrow -1/3$

$-\frac{1}{3}$ slope at $(-2,5)$

$y=mx+b$

$5=-\frac{1}{3}(-2)+b$

$5=\frac{2}{3}+b$

$\frac{13}{3}=b$

$y=-\frac{1}{3}x+\frac{13}{3}$

$3y=-x+13$

$\boxed{x+3y=13}$

(23) Circle tangent to y-axis at $y=3$ one intercept at $x=1$.

$(0,3)$ and $(1,0)$

midpoint $=\left(\frac{0+1}{2}, \frac{3+0}{2}\right)=\left(\frac{1}{2}, \frac{3}{2}\right)$

$m=\frac{0-3}{1-0}=-3$

$\perp m = 1/3$

$y=mx+b$

$\frac{3}{2}=\frac{1}{3}\left(\frac{1}{2}\right)+b$

$b=4/3$

$y=\frac{1}{3}x+4/3$ (goes through middle of circle)

$y=3$

$y=1/3x+4/3$ intersect at $(5,3)$

$(x-5)^2+(y-3)^2=r^2$

$\boxed{(x-5)^2+(y-3)^2=25}$

$r=\sqrt{(5-1)^2+(3-0)^2}=\sqrt{25}$

$r^2=25$

$|x-5|=4$

$x=1$ OR 9

intercept $x=9$
$x=1$

24) $P(x, y)$

$A(-1, 1)$

$B(2, -1)$

$3\overline{PB} = PA$

$$3\left(\sqrt{(x-2)^2+(y+1)^2}\right) = \sqrt{(x+1)^2+(y-1)^2}$$

$$9\left[(x-2)^2+(y+1)^2\right] = (x+1)^2+(y-1)^2$$

$$9\left[x^2-4x+4+y^2+2y+1\right] = x^2+2x+1+y^2-2y+1$$

$$9\cancel{x^2}-3\cancel{6}x+3\cancel{6}+9\cancel{y^2}+18\cancel{y}+\cancel{9} = \cancel{x^2}+\cancel{2x}+\cancel{1}+\cancel{y^2}-\cancel{2y}+\cancel{1}$$

$$\boxed{8x^2-38x+8y^2+20y+43=0}$$

(25) a) $\dfrac{3x+1}{\sqrt{x^2+x-2}}$

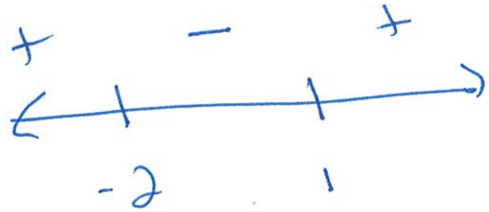
Domain

$x^2+x-2>0$

$(x+2)(x-1)>0$

critical points

$-2, 1$



$\boxed{x<-2 \text{ or } x>1}$

b) i) $f(x)=7$

$\boxed{\begin{array}{l}\text{Domain: All real \#'s} \\ \text{Range: } 7\end{array}}$

ii) $g(x)=\dfrac{5x-3}{2x+1}$

Domain: $2x+1 \neq 0$

$x \neq -1/2$

$\boxed{\begin{array}{l}\text{All real \#'s} \\ \text{except } -1/2\end{array}}$

Range: $\dfrac{5x-3}{2x-1}$

$\boxed{\begin{array}{l}\text{All real \#'s} \\ \text{except } 5/2\end{array}}$

(26) $f(x) = \frac{|x|}{x}$ show $f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$

Find domain & Range

$x > 0 \rightarrow \frac{|10|}{10} = 1, \quad \frac{|28|}{28} = 1$

$x < 0 \rightarrow \frac{|-10|}{-10} = -1, \quad \frac{|-28|}{-28} = -1$

Domain: All real #'s except 0

Range: $\{1, -1\}$

27 $\dfrac{f(x+h)-f(x)}{h}$

a) $f(x)=2x+3$

$$\frac{2(x+h)+3-(2x+3)}{h}$$

$$\frac{\cancel{2x}+2h+\cancel{3}-\cancel{2x}-\cancel{3}}{h}=\frac{2h}{h}=\boxed{2}$$

b) $f(x)=\dfrac{1}{x+1}$

$$\frac{\dfrac{1}{(x+h)+1}-\dfrac{1}{x+1}}{h}=\frac{(x+1)-(x+h+1)}{h(x+h+1)(x+1)}$$

$$=\frac{\cancel{x}+\cancel{1}-\cancel{x}-\cancel{h}^{-1}-\cancel{1}}{\cancel{h}(x+h+1)(x+1)}$$

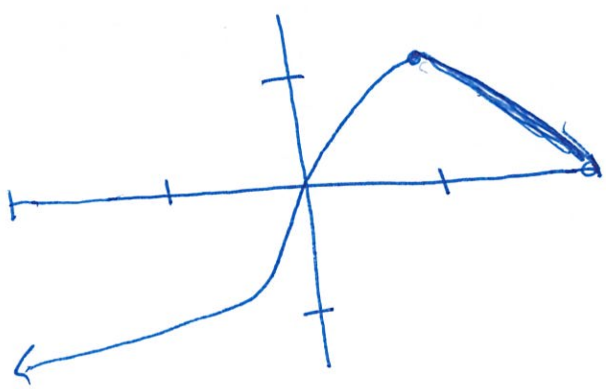
$$\boxed{\frac{-1}{(x+1)(x+h+1)}}$$

27 c) $f(x) = x^2$

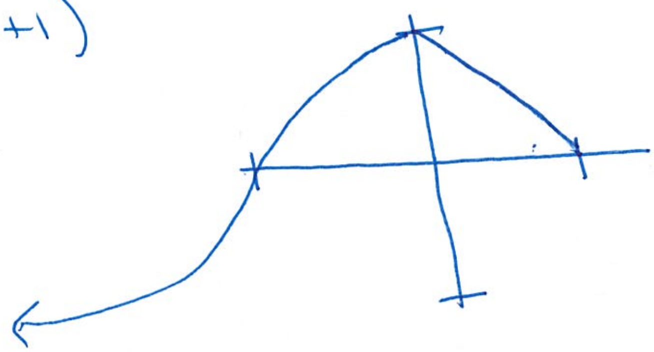
$$\frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \frac{(2x+h)h}{h} = \boxed{2x+h}$$

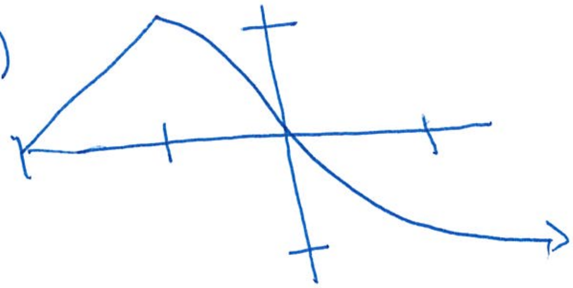
28



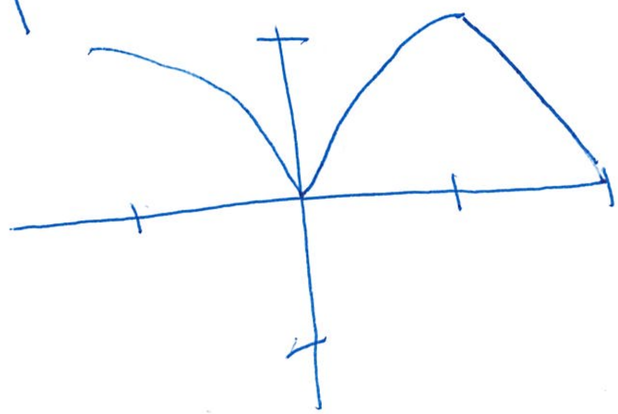
a) $f(x+1)$



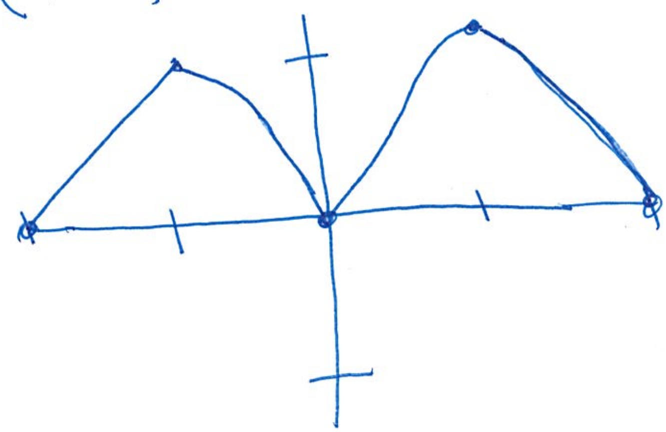
28 b) $f(-x)$



c) $|f(x)|$

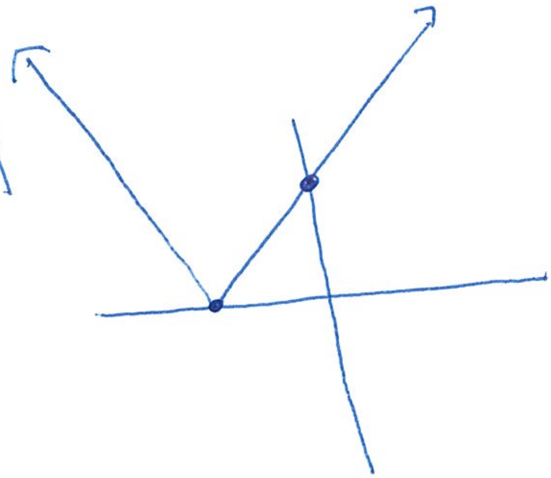


d) $f(|x|)$

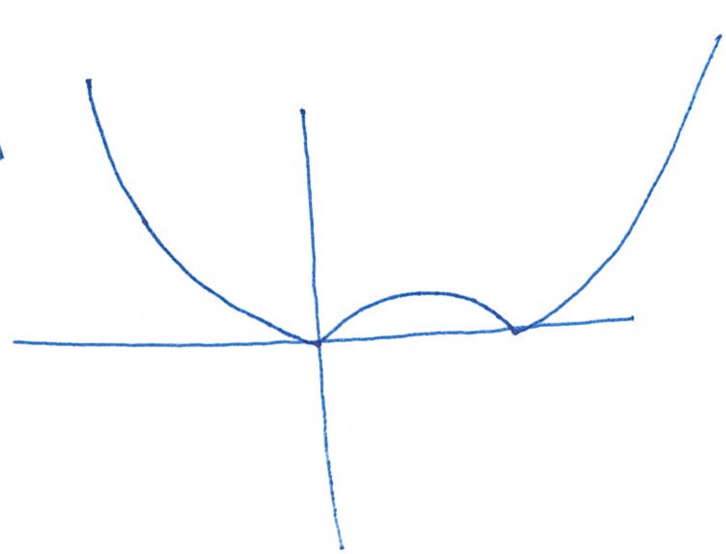


29) $g(x) = |3x+2|$

a)



b) $|x(x-1)|$



30 a) x-int $(-1,0),(3,0)$

Range $y \le 4$

midpoint $\left(\frac{3+(-1)}{2}, \frac{0+0}{2}\right) = (1, 0)$

vertex line

$x = 1$

*Since range was ≤ 4, vertex $(1,4)$

$(x-h)^2 = 4a(y-k)$

$(-1-1)^2 = 4a(0-4)$

$4 = 4a(-4)$

$a = -1/4$

$(x-1)^2 = 4(-1/4)(y-4)$

$x^2 - 2x + 1 = -y + 4$

$x^2 - 2x - 3 = -y$

$\boxed{y = -x^2 + 2x + 3}$

(30) b) $y = 2x^2 - 4x + 3$

Find x-intercepts

$2x^2 - 4x + 3 = 0$

$b^2 - 4ac$ is negative, so no x-intercepts

$x = 0,\ y = 3 \quad (0,3)$

y-intercept

axis

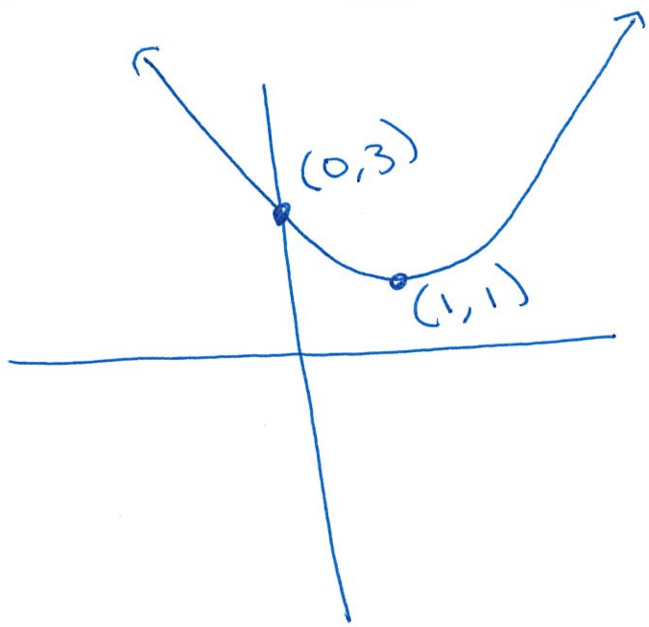
$\frac{-b}{2a} = \frac{4}{4} = 1$

vertex

$y = 2(1)^2 - 4(1) + 3$

$y = 1$

$(1,1)$



31) a) $\begin{cases} x = t+1 \\ y = t^2 - t \end{cases}$

$t = x - 1$

$y = (x-1)^2 - (x-1)$

$\boxed{y = x^2 - 3x + 2}$

b) $\begin{cases} x = \sqrt[3]{t} - 1 \\ y = t^2 - t \end{cases} \longrightarrow (x+1)^3 = t$

$\downarrow$

$y = \left((x+1)^3\right)^2 - (x+1)^3$

$y = (x+1)^3\left((x+1)^3 - 1\right)$

$y = (x+1)^3\left(x^3 + 3x^2 + 3x + \cancel{1} - \cancel{1}\right)$

$\boxed{y = (x+1)^3(x)(x^2 + 3x + 3)}$

31 c) $\begin{cases} x = \sin t \\ y = \cos t \end{cases}$

$$\sin^2 t + \cos^2 t = 1$$

$$\boxed{x^2 + y^2 = 1}$$

(32) a) $f(x) = 2x+3$

Inverse

$x = 2y+3$

$y = \frac{x-3}{2}$ $\boxed{f^{-1}(x) = \frac{x-3}{2}}$

b) $f(x) = \frac{x+2}{5x-1}$

$x = \frac{y+2}{5y-1}$

$5xy - x = y+2$

$5xy - y = x+2$

$y(5x-1) = x+2$

$y = \frac{x+2}{5x-1}$ $\boxed{f^{-1}(x) = \frac{x+2}{5x+1}}$

32 c) $f(x) = x^2 + 2x - 1$

$$x = y^2 + 2y - 1$$

rewrite

$$x = y^2 + 2y + 1 - 2$$

$$x = (y+1)^2 - 2$$

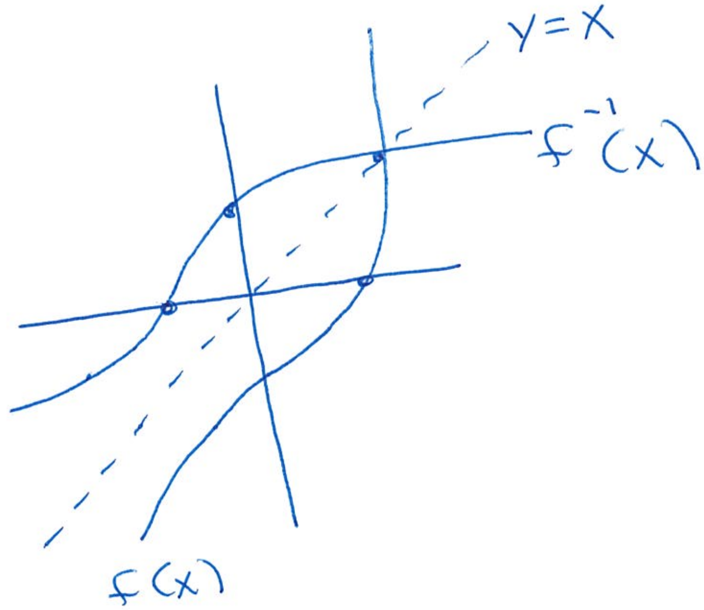
$$x + 2 = (y+1)^2$$

$$\sqrt{x+2} = y + 1$$

$$\boxed{y = \sqrt{x+2} - 1}$$

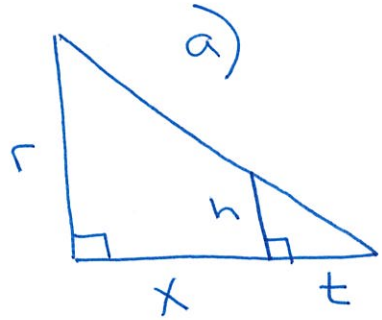
33 Sketch inverse

* swith x & y



34

a)



$$\frac{x+t}{t} = \frac{r}{h}$$

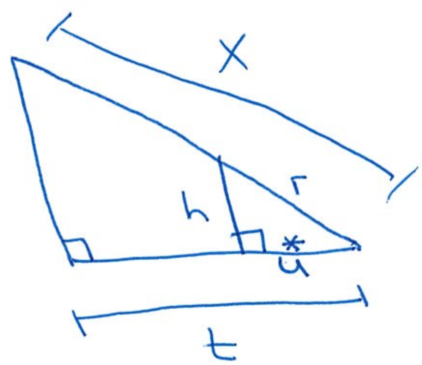
$$x+t = \frac{rt}{h}$$

$$x = \frac{rt}{h} - t$$

$$x = t\left(\frac{r}{h} - 1\right)$$

$$\boxed{x = t\left(\frac{r-h}{h}\right)}$$

b)



$$h^2 + u^2 = r^2$$

$$u^2 = r^2 - h^2$$

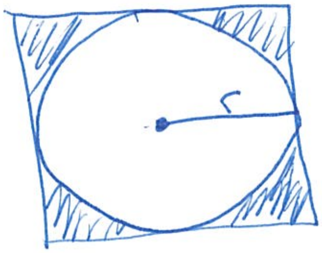
$$u = \sqrt{r^2 - h^2}$$

$$\frac{x}{t} = \frac{r}{\sqrt{r^2-h^2}}$$

$$\boxed{x = \frac{rt}{\sqrt{r^2-h^2}}}$$

35

a)



$$A_{square} = (2r)^2 = 4r^2$$

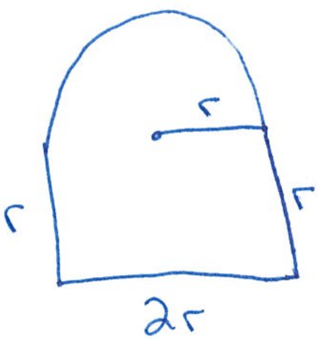
$$A_{circle} = \pi r^2$$

$$A_{shaded} = 4r^2 - \pi r^2$$

$$\text{ratio} = \frac{4r^2 - \pi r^2}{4r^2} = \frac{\cancel{r^2}(4-\pi)}{4\cancel{r^2}} = \frac{4-\pi}{4}$$

$$\boxed{1 - \frac{\pi}{4}}$$

b)



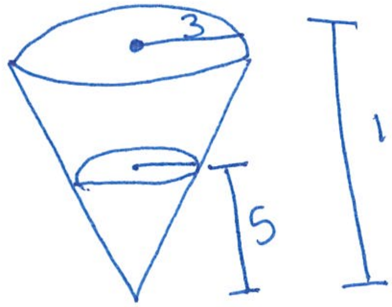
perimeter

$$P_{circle} = 2\pi r$$

$$P_{halfcircle} = \pi r$$

$$\text{Perimeter} = \boxed{4r + \pi r}$$

35) c)



$$\frac{3}{r} = \frac{10}{5}$$

$r = 1.5$

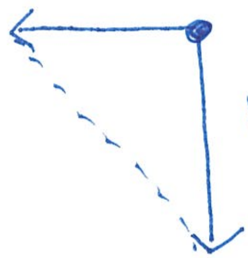
$r = \frac{3}{2}$

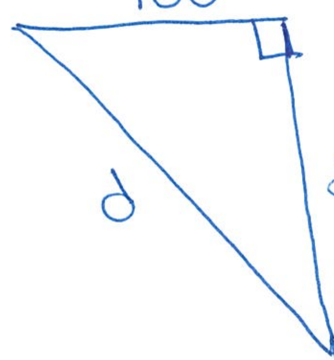
$A = \pi r^2$

$A = \pi \left(\frac{3}{2}\right)^2$

$$\boxed{A = \frac{9\pi}{4}}$$

d)



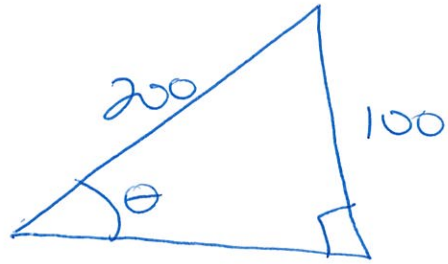


$100^2 + 200^2 = d^2$

$d = \sqrt{50{,}000}$

$$\boxed{d = 100\sqrt{5}}$$

(35) e)



$\sin\theta = \frac{100}{200}$

$\sin\theta = 1/2$

$\boxed{\theta = 30^\circ \text{ or } \pi/6}$

(36) memorize Formulas

for trig functions

Summary of Trigonometric Function Properties

Reciprocal Properties

$\cot x = \dfrac{1}{\tan x}$ or $\tan x \cot x = 1$

$\sec x = \dfrac{1}{\cos x}$ or $\cos x \sec x = 1$

$\csc x = \dfrac{1}{\sin x}$ or $\sin x \csc x = 1$

Quotient Properties

$\tan x = \dfrac{\sin x}{\cos x} = \dfrac{\sec x}{\csc x}$

$\cot x = \dfrac{\cos x}{\sin x} = \dfrac{\csc x}{\sec x}$

Pythagorean Properties

$\cos^2 x + \sin^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

Odd–Even Function Properties

$\sin(-x) = -\sin x$ (odd function)

$\cos(-x) = \cos x$ (even function)

$\tan(-x) = -\tan x$ (odd function)

$\cot(-x) = -\cot x$ (odd function)

$\sec(-x) = \sec x$ (even function)

$\csc(-x) = -\csc x$ (odd function)

Cofunction Properties

$\cos(90° - \theta) = \sin\theta$, $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\cot(90° - \theta) = \tan\theta$, $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

$\csc(90° - \theta) = \sec\theta$, $\csc\left(\frac{\pi}{2} - x\right) = \sec x$

Linear Combination of Cosine and Sine

$b\cos x + c\sin x = A\cos(x - D)$, where

$A = \sqrt{b^2 + c^2}$ and $D = \arctan\dfrac{c}{b}$

Composite Argument Properties

$\cos(A - B) = \cos A\cos B + \sin A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\sin(A - B) = \sin A\cos B - \cos A\sin B$

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

Sum and Product Properties

$2\cos A\cos B = \cos(A + B) + \cos(A - B)$

$2\sin A\sin B = -\cos(A + B) + \cos(A - B)$

$2\sin A\cos B = \sin(A + B) + \sin(A - B)$

$2\cos A\sin B = \sin(A + B) - \sin(A - B)$

$\cos x + \cos y = 2\cos\frac{1}{2}(x + y)\cos\frac{1}{2}(x - y)$

$\cos x - \cos y = -2\sin\frac{1}{2}(x + y)\sin\frac{1}{2}(x - y)$

$\sin x + \sin y = 2\sin\frac{1}{2}(x + y)\cos\frac{1}{2}(x - y)$

$\sin x - \sin y = 2\cos\frac{1}{2}(x + y)\sin\frac{1}{2}(x - y)$

Double Argument Properties

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

Half Argument Properties

$\sin\frac{1}{2}x = \pm\sqrt{\frac{1}{2}(1 - \cos x)}$

$\cos\frac{1}{2}x = \pm\sqrt{\frac{1}{2}(1 + \cos x)}$

$\tan\frac{1}{2}x = \pm\sqrt{\dfrac{1 - \cos x}{1 + \cos x}}$

$= \dfrac{\sin x}{1 + \cos x} = \dfrac{1 - \cos x}{\sin x}$