9.7 Mini Lecture

The Cross Product

Learning Objectives:

1. Find the cross product of two vectors
2. Know algebraic properties of the cross product
3. Know geometric properties of the cross product
4. Find a vector orthogonal to two given vectors
5. Find the area of a parallelogram

Examples:

1. If $v = 2i + 3j + 5k$ and $w = i + 2j + 3k$ Find $v \times w$

2. Use determinants to find the cross products in example #1.

3. Find a vector that is orthogonal to $u = 3i - 2j + k$ and $v = -i + 3j - k$

4. Find the area of a parallelogram whose vertices are P1 (0, 0, 0), P2 (3, -2, 1), P3 (-1, 3, -1), P4 (2,1,0).

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Cross product

$v = a_1 i + b_1 j + c_1 k$

$w = a_2 i + b_2 j + c_2 k$

$$v \times w = (b_1 c_2 - b_2 c_1) i - (a_1 c_2 - a_2 c_1) j + (a_1 b_2 - a_2 b_1) k$$

$u \times u = 0$

$u \times v = -(v \times u)$

$u \times v$ is orthogonal to both u and v

$\|u \times v\| = \|u\| \|v\| \sin\theta$

$\|u \times v\|$ is area of parallelogram with u and v as adjacent sides

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① $v = 2i + 3j + 5k$

$w = i + 2j + 3k$

$v \times w = [(3)(3) - (2)(5)]i - [(2)(3) - (1)(5)]j + [(2)(2) - (1)(3)]k$

$= \boxed{-i - j + k}$

② $v = 2i + 3j + 5k$

$w = i + 2j + 3k$

$$\begin{vmatrix} i & j & k \\ 2 & 3 & 5 \\ 1 & 2 & 3 \end{vmatrix}$$

$$\begin{vmatrix} 3 & 5 \\ 2 & 3 \end{vmatrix} i - \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} j + \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} k$$

$(9 - 10)i - (6 - 5)j + (4 - 3)k$

$\boxed{-i - j + k}$

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③ Find vector orthogonal to

$u = 3i - 2j + k$

and $v = -i + 3j - k$

* $u \times v$ is orthogonal to u and v

$$\begin{vmatrix} i & j & k \\ 3 & -2 & 1 \\ -1 & 3 & -1 \end{vmatrix}$$

$$(2-3)i - (-3+1)j + (9-2)k$$

$$\boxed{-i + 2j + 7k}$$

④ $P_1(0,0,0)$
$P_2(3,-2,1)$
$P_3(-1,3,-1)$
$P_4(2,1,0)$

Area of parallelogram

$\|u \times v\|$ adjacent sides

$$u = \overrightarrow{P_1P_2} = (3-0)i + (-2-0)j + (1-0)k$$

$$u = 3i - 2j + k$$

$$v = \overrightarrow{P_1P_3} = (-1-0)i + (3-0)j + (-1-0)k$$

$$v = -i + 3j - k$$

$$u \times v = -i + 2j + 7k$$

$$\text{Area} = \|u \times v\| = \sqrt{(-1)^2 + (2)^2 + (7)^2} = \sqrt{54} = \boxed{3\sqrt{6}}$$