

9.6 Mini Lecture

Vectors in Space

Learning Objectives:

1. Find the distance between two points in space
2. Find position vectors in space
3. Perform operations on vectors
4. Find the Dot Product
5. Find the angle between two vectors
6. Find the direction Angles of a vector

Examples:

1. Find the distance from $P1 = (0, -4, 3)$ to $P2 = (6, -5, -1)$.
2. Find the position vector starting at $P1$ (in #1) to $P2$ (in #1).
3. If $v = 3i + j - 2k$ and $w = -3i + 2j - k$; find the following
 - a. $-v + 2w$
 - b. $||v + w||$
 - c. $v \cdot w$ (Dot Product)
 - d. direction angle of v
 - e. angle between vectors

9.6 mini notes

$$P_1 = (x_1, y_1, z_1)$$

$$P_2 = (x_2, y_2, z_2)$$

Distance

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$V = ai + bj + ck$$

position
vector

$$V = \vec{P_1 P_2} = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$$

$$V \cdot W = a_1 a_2 + b_1 b_2 + c_1 c_2 \quad \text{dot product}$$

$$\cos \theta = \frac{U \cdot V}{\|U\| \|V\|}$$

$$\cos \alpha = \frac{a}{\|v\|}$$

$$\cos \beta = \frac{b}{\|v\|}$$

$$\cos \gamma = \frac{c}{\|v\|}$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$v = \|v\| [\cos \alpha i + \cos \beta j + \cos \gamma k]$$

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$$\textcircled{1} P_1 = (0, -4, 3)$$

$$P_2 = (6, -5, -1)$$

$$d = \sqrt{(6-0)^2 + (-5-(-4))^2 + (-1-3)^2}$$

$$d = \sqrt{53}$$

$\textcircled{2}$ position vector

$$P_1 = (0, -4, 3)$$

$$P_2 = (6, -5, -1)$$

$$V = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$$

$$V = (6-0)i + (-5-(-4))j + (-1-3)k$$

$$V = 6i - j - 4k$$

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$$\textcircled{3} \quad v = 3i + j - 2k$$

$$w = -3i + 2j - k$$

a) $-v + 2w$

$$-(3i + j - 2k) + 2(-3i + 2j - k) = \boxed{-9i + 3j}$$

b) $\|v + w\|$

$$\|3j - 3k\| = \sqrt{(3)^2 + (-3)^2} = \boxed{3\sqrt{2}}$$

c) $v \cdot w$ dot product

$$(3)(-3) + (1)(2) + (-2)(-1) = \boxed{-5}$$

d) direction angle of v

$$v = 3i + j - 2k$$

$$\|v\| = \sqrt{3^2 + 1^2 + (-2)^2} = \sqrt{14}$$

$$\cos \alpha = \frac{3}{\sqrt{14}} \rightarrow \alpha = \boxed{36.7^\circ}$$

$$\cos \beta = \frac{1}{\sqrt{14}} \rightarrow \beta = \boxed{74.5^\circ}$$

$$\cos \gamma = \frac{-2}{\sqrt{14}} \rightarrow \gamma = \boxed{122.3^\circ}$$

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③ e) angle between vectors

$$v = 3i + j - 2k$$

$$w = -3i + 2j - k$$

$$\cos \theta = \frac{v \cdot w}{\|v\| \|w\|} = \frac{3(-3) + 1(2) + (-2)(-1)}{\sqrt{14} \sqrt{14}}$$

$$\cos \theta = \frac{-5}{14}$$

$$\theta = 110.9^\circ$$