Mini-Lecture 9.4

Vectors

Learning Objectives:

1. Graph Vectors (p. 595)
2. Find a Position Vector (p. 596)
3. Add and Subtract Vectors Algebraically (p. 597)
4. Find a Scalar Multiple and the Magnitude of a Vector (p. 598)
5. Find a Unit Vector (p. 598)
6. Find a Vector from Its Direction and Magnitude (p. 599)
7. Model with Vectors (p. 600)

Examples:

1. Let $\mathbf{v}$ be a vector from initial point $P_1(-4,-2)$ to terminal point $P_2(-1,5)$. Write $\mathbf{v}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

2. Find the unit vector in the same direction as $\mathbf{v} = 2\mathbf{i} - 7\mathbf{j}$.

3. If $\mathbf{u} = 3\mathbf{i} - 6\mathbf{j}$ and $\mathbf{v} = -2\mathbf{i} + \mathbf{j}$, find $\|\mathbf{u}-\mathbf{v}\|^2 + \|\mathbf{u}+\mathbf{v}\|^2$.

4. A small plane is flying at a speed of 200 miles per hour on a bearing of $N55^\circ E$. The wind is blowing from west to east at 35 miles per hour. Approximate the ground speed of the plane to the nearest mile per hour. Approximate the true course of the plane to the nearest tenth of a degree.

9.4 mini lecture

Vectors $v+w=w+v$ commutative

$u+(v+w)=(u+v)+w$ associative

$v+0=0+v=v$

$v+(-v)=0$

magnitude of vector $\|v\|$

$\|v\| \geq 0$

$\|v\|=0$ if and only if $v=0$

$\|-v\|=\|v\|$

$\|\alpha v\|=|\alpha|\|v\|$

If v is a vector and α is scalar

Unit vector

$\|u\|=1$

9.4 mini lecture continued

algebraic vector v

$$v = \langle a, b \rangle$$

$P_1(x_1, y_1)$, $P_2(x_2, y_2)$ $\Rightarrow$ $v = \overrightarrow{P_1P_2}$ $\qquad v = \langle x_2 - x_1, y_2 - y_1 \rangle$

$$v = a_1 i + b_1 j = \langle a_1, b_1 \rangle$$

$$w = a_2 i + b_2 j = \langle a_2, b_2 \rangle$$

$$v + w = (a_1 + a_2) i + (b_1 + b_2) j = \langle a_1 + a_2, b_1 + b_2 \rangle$$

$$v - w = (a_1 - a_2) i + (b_1 - b_2) j = \langle a_1 - a_2, b_1 - b_2 \rangle$$

$$\alpha v = (\alpha a_1) i + (\alpha b_1) j = \langle \alpha a_1, \alpha b_1 \rangle$$

$$\|v\| = \sqrt{a_1^2 + b_1^2}$$

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Unit vector

$$u = \frac{v}{\|v\|}$$

$$v = \|v\| u$$

$$v = \|v\| (\cos\alpha i + \sin\alpha j)$$

9.4 mini lecture continued

① $P_1(-4,-2)$, $P_2(-1,5)$

$$v = \langle x_2 - x_1, y_2 - y_1 \rangle$$

$$= \langle -1-(-4), 5-(-2) \rangle$$

$$= \langle 3, 7 \rangle$$

$$= \boxed{3i + 7j}$$

② $v = 2i - 7j$

$$u = \frac{v}{\|v\|} = \frac{2i-7j}{\sqrt{2^2+(-7)^2}} = \frac{2i-7j}{\sqrt{53}}$$

$$= \boxed{\frac{2\sqrt{53}}{53}i - \frac{7\sqrt{53}}{53}j}$$

9.4 mini lecture continued

③ $u = 3i - 6j$

$v = -2i + j$

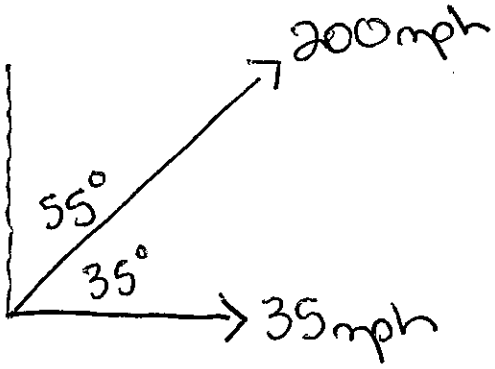
find $\|u-v\|^2 + \|u+v\|^2$

$\|5i - 7j\|^2 + \|i - 9j\|^2$

$\left(\sqrt{5^2+(-7)^2}\right)^2 + \left(\sqrt{(1)^2+(-5)^2}\right)^2$

$74 + 26 = \boxed{100}$

(4)



$V_a = 200(\cos 35\,i + \sin 35\,j)$

$V_a = 163.83i + 114.72j$

$V_w = 35(\cos 0\,i + \sin 0\,j)$

$V_w = 35i$

$V_g = V_a + V_w = 163.83i + 114.72j + 35i$

$V_g = 198.83i + 114.72j$

$\|V_g\| = \sqrt{198.83^2 + 114.72^2} = 230\text{mph}$

$\tan\theta = \frac{y}{x} = \frac{114.72}{198.83} = 30°$

N60°E