Mini-Lecture 9.3
The Complex Plane; De Moivre's Theorem

Learning Objectives:

1. Plot Points in the Complex Plane (p. 585)
2. Convert a Complex Number between Rectangular Form and Polar Form (p. 586)
3. Find Products and Quotients of Complex Numbers in Polar Form (p. 587)
4. Use De Moivre's Theorem (p. 588)
5. Find Complex Roots (p. 589)

Examples:

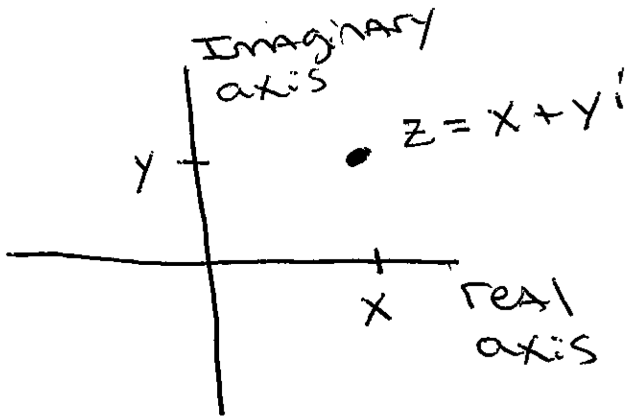
1. Write $-4+5i$ in polar form. Express θ in degrees.

2. Write $15\left(\cos 330^{\circ}+i \sin 330^{\circ}\right)$ in rectangular form.

3. If $z_1=8\left(\cos \frac{3\pi}{8}+i \sin \frac{3\pi}{8}\right)$ and $z_2=7\left(\cos \frac{5\pi}{8}+i \sin \frac{5\pi}{8}\right)$, find $z_1 z_2$. Leave the answer in polar form.

4. Find the indicated power: $(-2-2i)^{10}$.

9.3 mini lecture



magnitude $|z| = \sqrt{x^2 + y^2}$

$\bar{z}$ is conjugate $= x - yi$

$|z| = \sqrt{z\bar{z}}$

$$z = x + yi = (r\cos\theta) + (r\sin\theta)i$$
$$= r(\cos\theta + i\sin\theta)$$

$|z| = r$

9.3 mini lecture continued

$$z_1 z_2 = r_1 r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

* [redacted] $[0, 360]$

$$z^n = r^n\left(\cos(n\theta) + i\sin(n\theta)\right)$$

$$z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta_0}{n} + \frac{2\pi k}{n}\right) + i\sin\left(\frac{\theta_0}{n} + \frac{2\pi k}{n}\right)\right]$$

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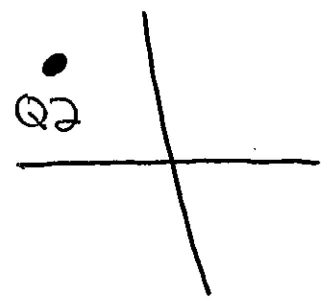
① write $-4+5i$ in polar form



$r = \sqrt{x^2+y^2} = \sqrt{(-4)^2+(5)^2} = \sqrt{41}$

$\tan\theta = \frac{5}{-4} \longrightarrow \theta = -51.34$
Q4

$180 - 51.34 = \underline{\underline{128.66}}$ Q2

$z = r(\cos\theta + i\sin\theta)$

$= \boxed{\sqrt{41}\,(\cos 128.7 + i\sin 128.7)}$

9.3 mini lecture continued

(2) write $15(\cos 330° + i \sin 330°)$ in rectangular form

$15 \cos 330° + 15(i)(\sin 330°)$

$15\left(\frac{\sqrt{3}}{2}\right) + 15i\left(-\frac{1}{2}\right)$

$$\boxed{\frac{15\sqrt{3}}{2} - \frac{15}{2}i}$$

(3) $z_1 = 8\left(\cos \frac{3\pi}{8} + i \sin \frac{3\pi}{8}\right)$

$z_2 = 7\left(\cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8}\right)$

$$z_1 z_2 = r_1 r_2 \left[\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)\right]$$

$$= (8)(7)\left[\cos\left(\frac{3\pi}{8} + \frac{5\pi}{8}\right) + i \sin\left(\frac{3\pi}{8} + \frac{5\pi}{8}\right)\right]$$

$$= \boxed{56\left[\cos \pi + i \sin \pi\right]}$$

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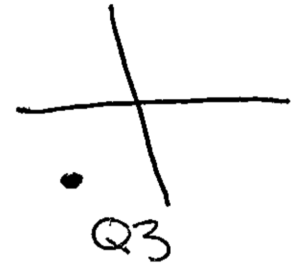
9.3 mini lecture continued

④ $(-2-2i)^{10}$



$$r=\sqrt{x^2+y^2}=\sqrt{(-2)^2+(-2)^2}$$

$$=2\sqrt{2}$$

$$\tan\theta=\frac{-2}{-2}\quad \theta=45^\circ \text{ Q1}$$

$$\underline{\underline{\theta=225^\circ}} \text{ Q3}$$

$$\underline{\underline{5\pi/4}}$$

$$z^n=r^n\left[\cos(n\theta)+i\sin(n\theta)\right]$$

$$=(2\sqrt{2})^{10}\left[\cos\left(10\cdot 5\pi/4\right)+i\sin\left(10\cdot 5\pi/4\right)\right]$$

$$=32768\left[0+i(1)\right]$$

$$=\boxed{0+32768i}$$