Mini-Lecture 9.2

Polar Equations and Graphs

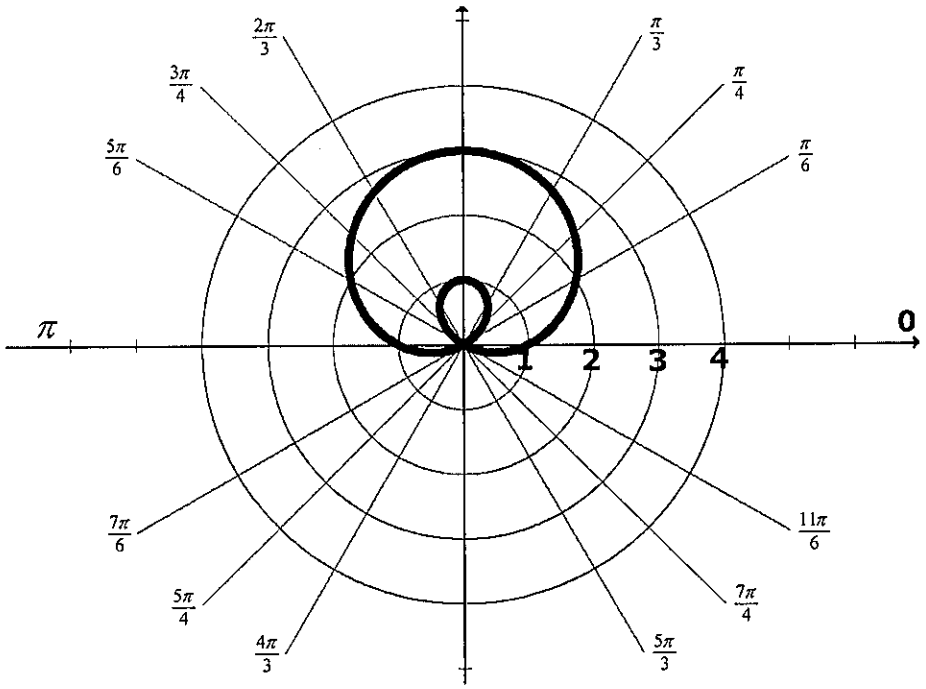
Learning Objectives:

1. Graph and Identify Polar Equations by Converting to Rectangular Equations (p. 569)
2. Graph Polar Equations Using a Graphing Utility (p. 571)
3. Test Polar Equations for Symmetry (p. 574)
4. Graph Polar Equations by Plotting Points (p. 575)

Examples:

1. Identify and graph: $r = \dfrac{3}{\sin\theta}$

2. The polar equation for the graph is either $r = a + b\cos\theta$ or $r = a + b\sin\theta$, $a > 0,\ b > 0$. Select the correct equation and find the values of a and b.



3. Identify and graph the polar equation. $r = 5\cos 2\theta$

4. Graph the polar equation by hand. $r = \dfrac{1}{2 - 2\cos\theta}$.

9.2 mini lecture

Symmetry

Polar Axis (x-Axis)

replace θ by $-\theta$

Line $\theta = \pi/2$ (y-axis)

replace θ by $\pi - \theta$

Pole (origin)

replace r by $-r$

9.2 mini lecture continued

Lines

$r\sin\theta = a$ horizontal

$a \geq 0$ above pole

$a < 0$ $|a|$ below pole

$r\cos\theta = a$ vertical

$a \geq 0$ right of pole

$a < 0$ $|a|$ left of pole

Circle

$r = 2a\sin\theta$ radius a, center $(0, a)$ rectangular

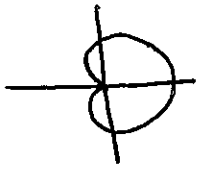
$r = -2a\sin\theta$ radius a, center $(0, -a)$ rectangular

$r = 2a\cos\theta$ radius a, center $(a, 0)$ rectangular

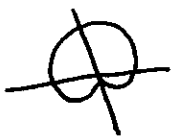
$r = -2a\cos\theta$ radius a, center $(-a, 0)$ rectangular

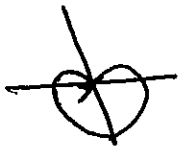
9.2 mini lecture continued

Cardioid $a>0$

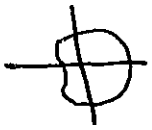
$r = a(1+\cos\theta)$ 

$r = a(1-\cos\theta)$ 

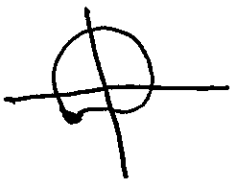
$r = a(1+\sin\theta)$ 

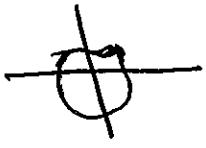
$r = a(1-\sin\theta)$ 

Limaçon without inner loop $a>0$ $b>0$ $a>b$

$r = a+b\cos\theta$ 

$r = a-b\cos\theta$ 

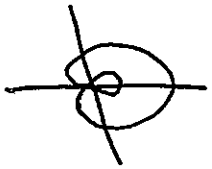
$r = a+b\sin\theta$ 

$r = a-b\sin\theta$ 

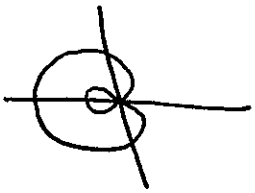
9.2 mini lecture continued

Limaçon with inner loop $a>0$, $b>0$, $a<b$

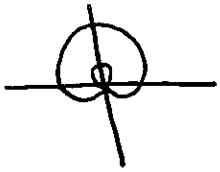
$r = a + b\cos\theta$



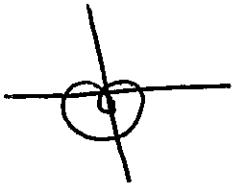
$r = a - b\cos\theta$



$r = a + b\sin\theta$



$r = a - b\sin\theta$



9.2 mini lecture continued

Rose n even, then $2n$ petals
n odd, then n petals $\quad a \neq 0$

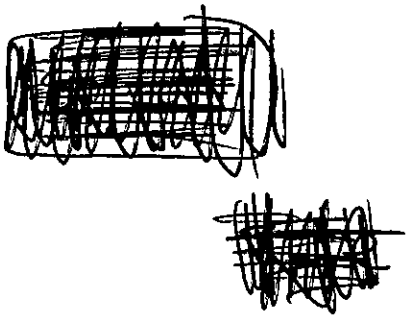
$$r = a\cos(n\theta)$$

$$r = a\sin(n\theta)$$

Lemniscate propeller shape

$$r^2 = a^2 \sin 2\theta$$

$$r^2 = a^2 \cos 2\theta$$

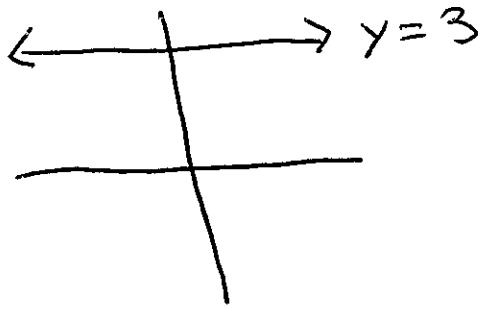


9.2
mini lecture continued

① $r = \frac{3}{\sin\theta}$

$r\sin\theta = 3$

$y = 3$



② $r = a + b\sin\theta$

contains $(1, 0)$

$1 = a + b\sin 0$

$1 = a + b(0)$

$a = 1$

contains $(3, \pi/2)$

$3 = 1 + b\sin(\pi/2)$

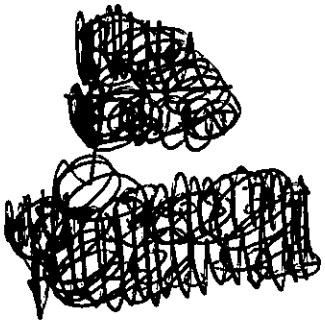
$3 = 1 + b(1)$

$b = 2$

$$r = 1 + 2\sin\theta$$

9.2 mini lecture continued

③ $r = 5\cos 2\theta$



* check symmetry

Polar Axis

$\theta \to -\theta$

$r = 5\cos(-2\theta)$

$* \cos\theta = \cos(-\theta)$

$r = 5\cos 2\theta$

symmetric polar axis

Line $\theta = \pi/2$

$\theta \to \pi - \theta$

$r = 5\cos(2(\pi - \theta))$

$r = 5\cos(2\pi - 2\theta)$

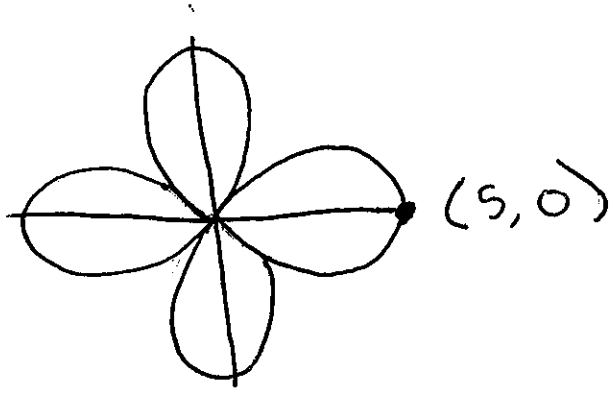
$r = 5[\cos 2\pi \cos 2\theta + \sin 2\pi \sin 2\theta]$

$r = 5[(1)(\cos 2\theta) + (0)(\sin 2\theta)]$

$r = 5\cos 2\theta$

symmetric line $\theta = \pi/2$

* Since symmetric with Polar axis and line $\theta = \pi/2$, then it is symmetric with Pole



9.2 mini lecture continued

④ $r = \dfrac{1}{2-2\cos\theta}$

* check symmetry

Polar axis

$\theta \to -\theta$

$\dfrac{1}{2-2\cos(-\theta)}$

$*\ \cos\theta = \cos(-\theta)$

$\dfrac{1}{2-2\cos\theta}$ ✓

Symmetric polar axis

Line $\theta = \pi/2$

$\theta \to (\pi - \theta)$

$\dfrac{1}{2-2\cos(\pi-\theta)}$

$\dfrac{1}{2-2(\cos\pi\cos\theta - \sin\pi\sin\theta)}$

$\dfrac{1}{2-2((-1)\cos\theta - (0)(\sin\theta))}$

$\dfrac{1}{2+2\cos\theta}$ ✗

Test Fails

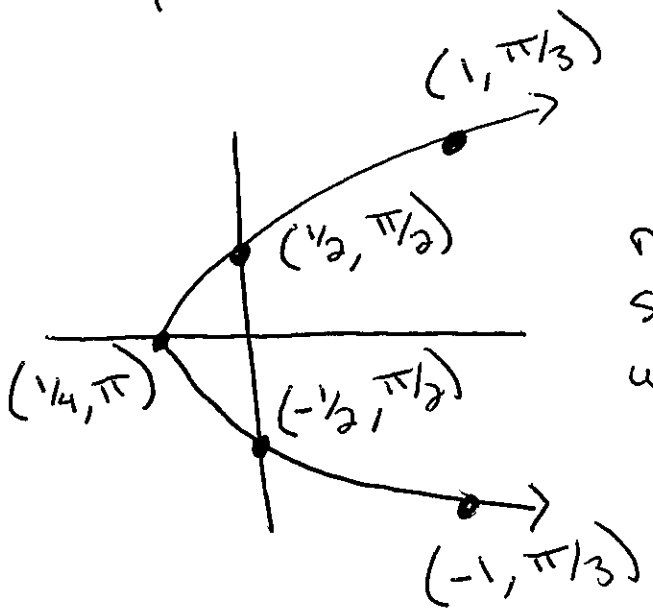
Pole

$r \to -r$

$-r = \dfrac{1}{2-2\cos\theta}$

$r = -\dfrac{1}{2-2\cos\theta}$ ✗

test Fails



mirror images since symmetry with polar axis