

Mini-Lecture 9.1

Polar Coordinates

Learning Objectives:

1. Plot Points Using Polar Coordinates (p. 560)
2. Convert from Polar Coordinates to Rectangular Coordinates (p. 562)
3. Convert from Rectangular Coordinates to Polar Coordinates (p. 564)
4. Transform Equations between Polar and Rectangular Form (p. 566)

Examples:

1. Find the rectangular coordinates of the polar point, $\left(-5, \frac{5\pi}{2}\right)$.
2. Find the polar coordinates of the rectangular point, $(-4, -4)$.
3. Convert $x^2 = 8y$ to a polar equation.
4. Convert $r = 4 \sin \theta + 6 \cos \theta$ to a rectangular equation.

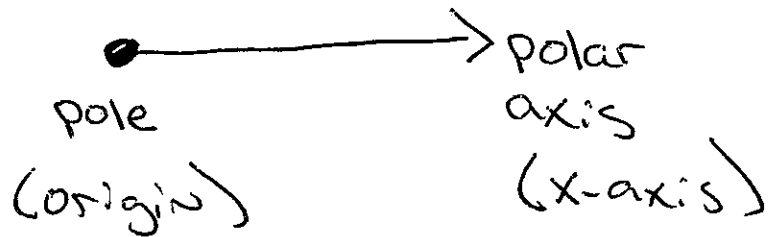
9.1 mini lecture

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$\tan \theta = \frac{y}{x}$$



① polar point $(-5, 5\pi/2)$

$$x = r \cos \theta$$

$$= -5 \cos(5\pi/2)$$

$$= -5 \cos(2\pi + \pi/2)$$

$$= -5 \cos(\pi/2)$$

$$= ~~0~~ -5(0) = 0$$

$$y = r \sin \theta$$

$$= -5 \sin(5\pi/2)$$

$$= -5 \sin(2\pi + \pi/2)$$

$$= -5 \sin(\pi/2)$$

$$= -5(1) = -5$$

rectangular
point
 $(0, -5)$

9.1 mini lecture continued

② rectangular point $(-4, -4)$

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-4)^2 + (-4)^2} = 4\sqrt{2}$$

$$\tan \theta = \frac{y}{x}$$

$$\tan \theta = \frac{-4}{-4}$$

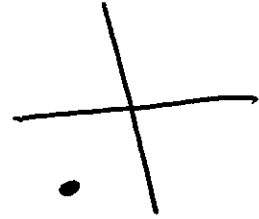
$$\tan \theta = 1$$

$$\theta = \tan^{-1}(1)$$

$$\theta = \pi/4 \quad \text{Q1}$$

$$\theta = \pi/4 + \pi = 5\pi/4 \quad \text{Q3}$$

$$\boxed{(4\sqrt{2}, 5\pi/4)} \text{ polar}$$



9.1 mini lecture continued

③ convert $x^2 = 8y$ to polar equation

$$x^2 = 8y$$

$$(r \cos \theta)^2 = 8(r \sin \theta)$$

$$r^2 \cos^2 \theta = 8r \sin \theta$$

* divide by r

$$r \cos^2 \theta = 8 \sin \theta$$

$$r = \frac{8 \sin \theta}{\cos^2 \theta}$$

9.1 mini lecture continued

④ convert $r = 4\sin\theta + 6\cos\theta$ to rectangular equation

$$r = 4\sin\theta + 6\cos\theta$$

$$r = 4\left(\frac{y}{r}\right) + 6\left(\frac{x}{r}\right)$$

$$r = \frac{4y}{r} + \frac{6x}{r}$$

$$r^2 = 4y + 6x$$

$$x^2 + y^2 = 4y + 6x$$

$$x^2 - 6x + y^2 - 4y = 0$$

$$x^2 - 6x + \boxed{9} + y^2 - 4y + \boxed{4} = 0 + \boxed{9} + \boxed{4}$$

$$\boxed{(x-3)^2 + (y-2)^2 = 13}$$