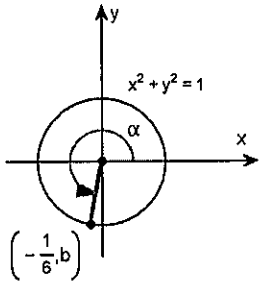
Student: _____ **Date:** _____	**Instructor:** Joe Betters **Course:** Pre-Calculus Pre AP (Master Course)	**Assignment:** 7.6 Classwork Day 2

1. Use a half-angle formula to find the exact value of **sin** $\left(\frac{\pi}{8}\right)$.

sin $\left(\frac{\pi}{8}\right)$ = _____

(Type an exact answer, using radicals as needed.)

2. Use the figure to evaluate the function given that f(x) = **sin** x.

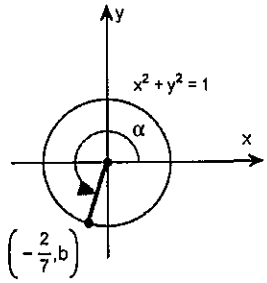


$f\left(\frac{\alpha}{2}\right)$

$f\left(\frac{\alpha}{2}\right)$ = _____

(Type an exact answer, using radicals as needed.)

3. Use the figure to evaluate the function given that h(x) = **tan** x.



$h\left(\frac{\alpha}{2}\right)$

$h\left(\frac{\alpha}{2}\right)$ = _____

(Type an exact answer, using radicals as needed.)

4. Establish the identity. $-\cos\theta = \dfrac{\tan^2\frac{\theta}{2} - 1}{\tan^2\frac{\theta}{2} + 1}$

Choose the sequence of steps below that verifies the identity.

○ **A.** $\dfrac{\tan^2\frac{\theta}{2} - 1}{\tan^2\frac{\theta}{2} + 1} = \dfrac{\frac{1-\cos\theta}{1+\cos\theta} - 1}{\frac{1-\cos\theta}{1+\cos\theta} + 1} = \dfrac{\frac{1-\cos\theta+1+\cos\theta}{1+\cos\theta}}{\frac{1-\cos\theta-(1+\cos\theta)}{1+\cos\theta}} = \dfrac{\frac{-2\cos\theta}{1+\cos\theta}}{\frac{2}{1+\cos\theta}} = -\cos\theta$

○ **B.** $\dfrac{\tan^2\frac{\theta}{2} - 1}{\tan^2\frac{\theta}{2} + 1} = \dfrac{\frac{1-\cos\theta}{1+\cos\theta} - 1}{\frac{1-\cos\theta}{1+\cos\theta} + 1} = \dfrac{\frac{1-\cos\theta-(1+\cos\theta)}{1+\cos\theta}}{\frac{1-\cos\theta+1+\cos\theta}{1+\cos\theta}} = \dfrac{\frac{-2\cos\theta}{1+\cos\theta}}{\frac{2}{1+\cos\theta}} = -\cos\theta$

○ **C.** $\dfrac{\tan^2\frac{\theta}{2} - 1}{\tan^2\frac{\theta}{2} + 1} = \dfrac{\frac{1+\cos\theta}{1-\cos\theta} - 1}{\frac{1+\cos\theta}{1-\cos\theta} + 1} = \dfrac{\frac{1-\cos\theta-(1+\cos\theta)}{1+\cos\theta}}{\frac{1-\cos\theta+1+\cos\theta}{1+\cos\theta}} = \dfrac{\frac{-2\cos\theta}{1+\cos\theta}}{\frac{2}{1+\cos\theta}} = -\cos\theta$

5. Solve the equation on the interval $0 \le \theta < 2\pi$.

$\cos(2\theta) + 14\sin^2\theta = 10$

What is the solution in the interval $0 \le \theta < 2\pi$? Select the correct choice and fill in any answer boxes in your choice below.

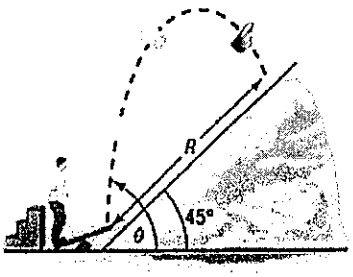
○ **A.** $\theta = \{\quad\}$
(Simplify your answer. Type an exact answer, using π as needed. Type your answer in radians. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

○ **B.** There is no solution.

6.

An object is propelled upward at an angle θ, $45° < \theta < 90°$, to the horizontal with an initial velocity of v_0 feet per second from the base of a plane that makes an angle of $45°$ with the horizontal. See the illustration. If air resistance is ignored, the distance R that it travels up the inclined plane is given by the function

$$R(\theta) = \frac{v_0^2\sqrt{2}}{16}\cos\theta(\sin\theta - \cos\theta).$$



(a) Using double-angle identities, determine which of the following equations is equivalent to the one given in the problem statement.

○ **A.** $R(\theta) = \frac{v_0^2\sqrt{2}}{32}[\sin(2\theta) - \cos(2\theta) - 1]$

○ **B.** $R(\theta) = \frac{v_0^2\sqrt{2}}{32}[\tan(2\theta) - \cos(2\theta) - 1]$

○ **C.** $R(\theta) = \frac{v_0^2\sqrt{2}}{32}[\cos(2\theta) - \sin(2\theta) - 1]$

(b) The angle θ that maximizes R, satisfies this equation $\sin(2\theta) + \cos(2\theta) = 0$. Solve this equation for θ.

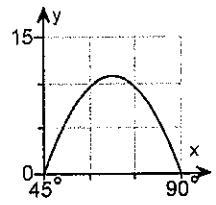
_____ °

(c) What is the maximum distance R if $v_0 = 28$ feet per second?

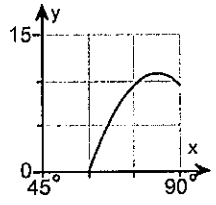
_____ feet (Round to the nearest hundredth as needed.)

(d) Graph $R = R(\theta)$, $45° < \theta < 90°$, and find the angle θ that maximizes the distance R. Also find the maximum distance. Use $v_0 = 28$ feet per second. Compare the results with the answers found earlier. Choose the correct graph below.

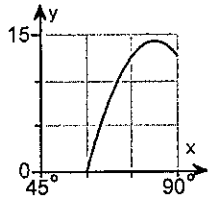
○ **A.**



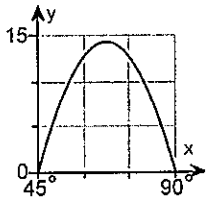
○ **B.**



○ **C.**



○ **D.**



The graph indicates that the angle θ that maximizes the distance R is _____ ° and the maximum distance is _____ feet. (Round to the nearest hundredth as needed.)

These results (1) _____ the answers in parts (b) and (c).

(1) ○ do not match
○ match

1. $\frac{\sqrt{2-\sqrt{2}}}{2}$

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2. $\frac{1}{2}\sqrt{\frac{7}{3}}$

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3. $-\frac{3\sqrt{5}}{5}$

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4. B. $\frac{\tan^2\frac{\theta}{2}-1}{\tan^2\frac{\theta}{2}+1} = \frac{\frac{1-\cos\theta}{1+\cos\theta}-1}{\frac{1-\cos\theta}{1+\cos\theta}+1} = \frac{\frac{1-\cos\theta-(1+\cos\theta)}{1+\cos\theta}}{\frac{1-\cos\theta+1+\cos\theta}{1+\cos\theta}} = \frac{\frac{-2\cos\theta}{1+\cos\theta}}{\frac{2}{1+\cos\theta}} = -\cos\theta$

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5. A. $\theta = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

(Simplify your answer. Type an exact answer, using π as needed. Type your answer in radians. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

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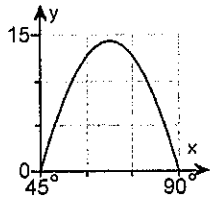
6. A. $R(\theta) = \frac{v_0^2\sqrt{2}}{32}[\sin(2\theta) - \cos(2\theta) - 1]$

67.5

14.35

D. 

67.5

14.35

(1) match

7.6 classwork day 2

① $\sin(\pi/8)$

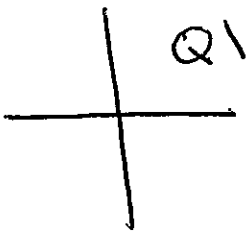
*use half angle

$$\sin\left(\frac{\pi/4}{2}\right) = \pm\sqrt{\frac{1}{2}(1-\cos x)}$$

$$= \pm\sqrt{\frac{1}{2}\left(1-\frac{\sqrt{2}}{2}\right)}$$

$$= \pm\sqrt{\frac{1}{2}-\frac{\sqrt{2}}{4}}$$

$$= \pm\frac{\sqrt{2-\sqrt{2}}}{2}$$

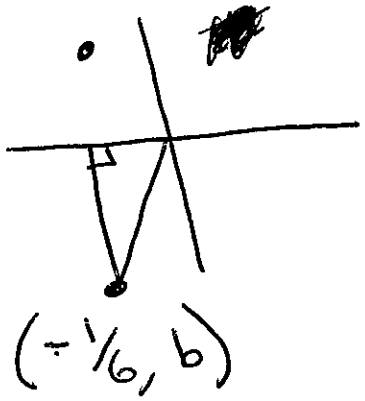


$$= \frac{\sqrt{2-\sqrt{2}}}{2}$$

7.6 classwork day 2 continued

② $f\left(\frac{\alpha}{2}\right)$

$f(x) = \sin x$



$x^2 + y^2 = 1$

$(-1/6)^2 + y^2 = 1$

$y^2 = \frac{35}{36}$

$y = -\frac{\sqrt{35}}{6}$

Q3

$\sin x = -\frac{\sqrt{35}}{6}$

$\cos x = -1/6$

$\tan x = \sqrt{35}$

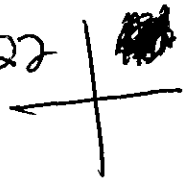
$$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1}{2}(1-\cos x)}$$

$$= \pm\sqrt{\frac{1}{2} - \frac{1}{2}(-1/6)}$$

$$= \pm\sqrt{\frac{1}{2} + \frac{1}{12}}$$

$$= \pm\sqrt{7/12}$$

Q2

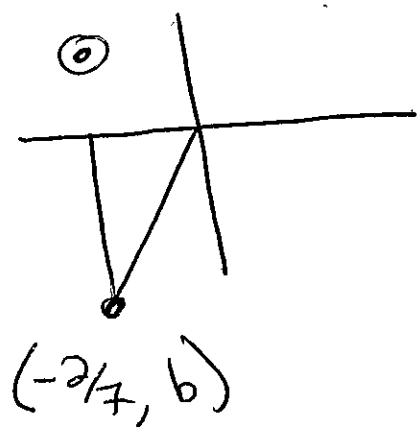


$$= \frac{1}{2}\frac{\sqrt{7}}{\sqrt{3}} = \boxed{\frac{\sqrt{21}}{6}}$$

7.6 classwork day 2 continued

③ $h(x) = \tan x$

$h\left(\frac{\alpha}{2}\right)$



$x^2 + y^2 = 1$

$\left(-\frac{2}{7}\right)^2 + y^2 = 1$

$y^2 = \frac{45}{49}$

$y = \pm \frac{3\sqrt{5}}{7}$

Q3 $y = -\frac{3\sqrt{5}}{7}$

$\sin x = -\frac{3\sqrt{5}}{7}$

$\cos x = -2/7$

$\tan\frac{\alpha}{2} = \frac{\sin x}{1+\cos x}$

$= \frac{-\frac{3\sqrt{5}}{7}}{1+(-2/7)}$

$= \frac{-\frac{3\sqrt{5}}{7}}{5/7} = \boxed{\frac{-3\sqrt{5}}{5}}$ Q2

7.6 day 2 classwork continued

(4) $-\cos\theta = \dfrac{\tan^2\left(\frac{\theta}{2}\right) - 1}{\tan^2\left(\frac{\theta}{2}\right) + 1}$

*use half angle $= \dfrac{\frac{1-\cos\theta}{1+\cos\theta} - 1}{\frac{1-\cos\theta}{1+\cos\theta} + 1}$

*common denominator $= \dfrac{\frac{1-\cos\theta-(1+\cos\theta)}{1+\cos\theta}}{\frac{1-\cos\theta+(1+\cos\theta)}{1+\cos\theta}}$

$= \dfrac{-2\cos\theta}{2}$

$= \boxed{-\cos\theta}$

B

7.6 classwork day 2 continued

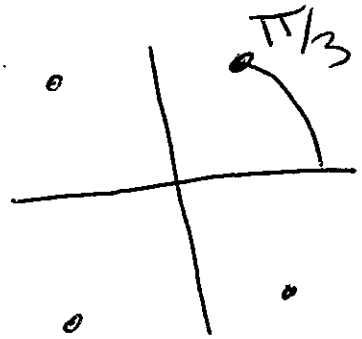
⑤ $\cos 2\theta + 14\sin^2\theta = 10$

$(1 - 2\sin^2\theta) + 14\sin^2\theta - 10 = 0$

$12\sin^2\theta - 9 = 0$

$\sin^2\theta = \frac{3}{4}$

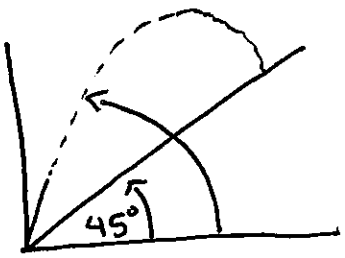
$\sin\theta = \pm\frac{\sqrt{3}}{2}$



$\theta = \left\{\pi/3, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

7.6 classwork day 2 continued

(6)



$$R(\theta) = \frac{V_0^2\sqrt{2}}{16}\cos\theta(\sin\theta - \cos\theta)$$

a) $\frac{V_0^2\sqrt{2}}{16}\sin\theta\cos\theta - \frac{V_0^2\sqrt{2}}{16}\cos^2\theta$ * distribute

$\frac{V_0^2\sqrt{2}}{16}\left(\frac{\sin 2\theta}{2}\right) - \frac{V_0^2\sqrt{2}}{16}\left(\frac{1+\cos 2\theta}{2}\right)$ * double angle

$\frac{V_0^2\sqrt{2}}{32}(\sin 2\theta - \cos 2\theta - 1)$ * factor out

A

7.6 Classwork Day 2 continued

6) b) $\sin 2\theta + \cos 2\theta = 0$

* divide by $\frac{1}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}}\sin 2\theta + \frac{1}{\sqrt{2}}\cos 2\theta = 0$$

* $\cos\phi = \frac{1}{\sqrt{2}}$, $\sin\phi = \frac{1}{\sqrt{2}}$, $\phi = \pi/4$ *

$$\sin 2\theta \cos\phi + \cos 2\theta \sin\phi = 0$$

* $\sin A\cos B + \cos A \sin B = \sin(A+B)$ *

$$\sin(2\theta + \phi) = 0$$

$$2\theta + \phi = 0 + K\pi$$

$$2\theta + \pi/4 = 0 + K\pi$$

$$2\theta = -\pi/4 + K\pi$$

$$\theta = -\pi/8 + \frac{\pi K}{2}$$

$$\theta = 3\pi/8 \quad Q1$$

$$\boxed{\theta = 67.5^\circ}$$

7.6 Classwork day 2 continued

6) c) max distance R if $V_0 = 28$ ft/sec

$$\frac{28^2\sqrt{2}}{32}\left(\sin(2\cdot 67.5^\circ) - \cos(2\cdot 67.5^\circ) - 1\right)$$

$$\frac{28^2\sqrt{2}}{32}\left(\sin 135^\circ - \cos 135^\circ - 1\right)$$

$$\frac{28^2\sqrt{2}}{32}\left(\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right) - 1\right)$$

14.35 feet

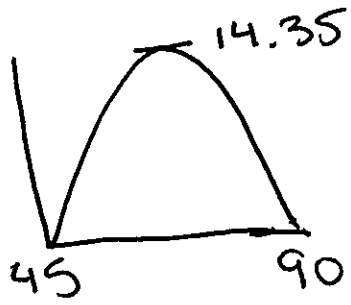
7.6 classwork day 2 continued

6) d) Graph $Y_1 = \frac{28^2\sqrt{2}}{32}(\sin 2x - \cos 2x - 1)$

* use maximum

67.5° and 14.35 max feet

Graph D



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