

Student: _____

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Date: _____

Course: Pre-Calculus Pre AP (Master Course)

Assignment: 6.3 Classwork Day 1

1. If $f(x) = \cos(x)$ and $f(a) = \frac{1}{3}$, find the exact value of the following.

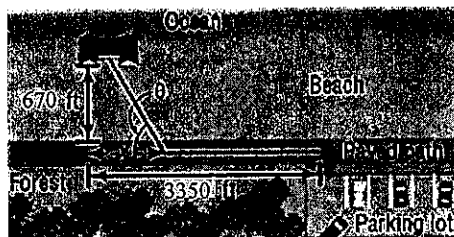
(a) $f(-a)$

(b) $f(a) + f(a + 2\pi) + f(a + 4\pi)$

(a) $f(-a) =$ _____ (Type the value as a reduced fraction.)

(b) $f(a) + f(a + 2\pi) + f(a + 4\pi) =$ _____ (Type the value as a reduced fraction.)

2.



From a parking lot you want to walk to a house on the ocean. The house is located 3350 ft down a paved path that parallels the beach, which is 670 ft wide. Along the path, you can walk 350 ft/min, but on the beach you can only walk 130 ft/min. Calculate the time T if you walk directly from the parking lot to the house.

The time T to get from the parking lot to the beach-house can be expressed as a function of the angle θ shown in the illustration and is equal to the following.

$$T(\theta) = \frac{67}{7} - \frac{67}{35 \tan \theta} + \frac{67}{13 \sin \theta}, \quad 0 < \theta < \frac{\pi}{2}$$

You must calculate the time T if you walk directly from the parking lot to the house. Use the following hint.

Hint: $\tan \theta = \frac{670}{3350}$.

$T =$ _____ minutes (Type an integer or a fraction with a radical if needed.)

3. Show that the period of $f(\theta) = \sin \theta$ is 2π .

If $f(\theta + p) = f(\theta)$ and p is the smallest such number then this smallest value is called the (fundamental) period of f . We will prove that the period of $f(\theta) = \sin \theta$ is 2π by contradiction. So, suppose that there is a number p other than 2π , such that

$f(\theta + p) = f(\theta)$, then $0 < p < 2\pi$.

Thus, $\sin(\theta + p) = \sin \theta$ for all θ . Let θ be 0.

$$\sin(\theta + p) = \sin \theta$$

$$\sin(0 + p) = \sin 0 \quad \text{Substitute 0 for } \theta.$$

$$\sin(p) = \sin 0 \quad \text{Simplify.}$$

For what value of p is the above equation true?

$$p = \underline{\hspace{2cm}}$$

Now, substitute $\frac{\pi}{2}$ for θ and the value found above for p in the equation $\sin(\theta + p) = \sin \theta$. (Simplify your answer.)

$$\sin(\theta + p) = \sin \theta$$

$$\sin\left(\underline{\hspace{2cm}}\right) = \sin \frac{\pi}{2} \quad \text{Add inside the parentheses.}$$

$$\underline{\hspace{2cm}} = \sin \frac{\pi}{2} \quad \text{Evaluate the expression on the left side.}$$

Thus, the assumption of existence of a number p other than 2π , such that $f(\theta + p) = f(\theta)$ and $0 < p < 2\pi$ results into the above equation. Is this equation true?

- No
 Yes

Does this prove that the period of $f(\theta) = \sin \theta$ is 2π ?

- A. Yes because it is proved that it is not possible to have a period value such that $0 < p < 2\pi$.
 B. No, this proves that the period of $f(\theta) = \sin \theta$ cannot be less than 2π . However there is a possibility that it can be greater than 2π .

- (1) $<$
 $>$

1. $\frac{1}{3}$

1

2. $\frac{67}{13}\sqrt{26}$

3. (1) <

0

 π

$\frac{3\pi}{2}$

-1

No

A. Yes because it is proved that it is not possible to have a period value such that $0 < p < 2\pi$.

6.3 classwork day 1

① $f(x) = \cos(x)$

$$f(a) = \frac{1}{3}$$

$$a) f(-a) = \cos(-1/3) = \boxed{\frac{1}{3}}$$

$$* \cos(-\theta) = \cos(\theta)$$

$$b) f(a) + f(a+2\pi) + f(a+4\pi)$$

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \boxed{1}$$

* add 2π to get same point on graph

② $\tan \theta = \frac{67}{7} - \frac{67}{35 \tan \theta} + \frac{67}{13 \sin \theta}$ $0 < \theta < \pi/2$

$$* \tan \theta = \frac{670}{3350} = \frac{y}{x} = \frac{1}{5}$$

$$x^2 + y^2 = r^2 = \frac{1}{5}$$

$$5^2 + 1^2 = r^2 \quad r = \sqrt{26}$$

$$* \sin \theta = \frac{1}{\sqrt{26}} = \frac{y}{r}$$

$$\frac{67}{7} - \frac{67}{35 \left(\frac{1}{5}\right)} + \frac{67}{13 \left(\frac{1}{\sqrt{26}}\right)} = \boxed{\frac{67\sqrt{26}}{13}}$$

6.3 classwork day 1 continued

③ Show period of $f(\theta) = \sin \theta$ is 2π

$f(\theta+p) = f(\theta)$, then $0 < p < 2\pi$

$\sin(\theta+p) = \sin \theta$ for all θ .

* Let θ be 0

$$\sin(\theta+p) = \sin \theta$$

$$\sin(0+p) = \sin 0$$

$$\sin p = 0$$

$$p = \pi$$

$$\sin \pi = 0$$

* Sub $\pi/2$ for θ

$$\sin(\theta+p) = \sin \theta$$

$$\sin(\pi/2 + \pi) = \sin \pi/2$$

$$\sin 3\pi/2 = \sin \pi/2$$

$$-1 = \sin \pi/2 \rightarrow \sin \pi/2 = 1$$

contradict

equation not true

A proved not possible to have a period value such that $0 < p < 2\pi$