Mini-Lecture 14.4

The Tangent Problem; The Derivative

Learning Objectives:

1. Find an Equation of the Tangent Line to the Graph of a Function (p. 891)
2. Find the Derivative of a Function (p. 893)
3. Find Instantaneous Rates of Change (p. 894)
4. Find the Instantaneous Speed of a Particle (p. 894)

Examples:

1. Find the slope of the tangent line to the graph of $f(x) = x - 2x^2$ at (-1, -3).

2. Find the derivative of $f(x) = \frac{3}{x}$ at (1, 3).

3. Use a graphing utility to find the derivative of $f(x) = x\cos x$ at $\frac{\pi}{4}$.

4. An explosion causes debris to rise vertically with an initial speed of 80 feet per second. The function, $s(t) = -16t^2 + 80t$, describes the height of the debris above the ground, $s(t)$, in feet, t seconds after the explosion. a. What is the instantaneous speed of the debris 1.5 seconds after the explosion? b. What is the instantaneous speed of the debris when it hits the ground?

14.4 mini notes

① $f(x) = x - 2x^2$ at $(c, f(c))$ $(-1, -3)$

$$\frac{f(x) - f(\text{limit})}{x - (\text{limit})} = \frac{f(x) - f(c)}{x - c}$$

$$\frac{(x - 2x^2) - (-3)}{x - (-1)}$$

* factor *

$$\frac{x - 2x^2 + 3}{x + 1} = \frac{-2x^2 + x + 3}{x + 1}$$

$$= \frac{-(2x^2 - x - 3)}{x + 1}$$

$$= \frac{-(2x - 3)(x + 1)}{(x + 1)}$$

$$= -(2(-1) - 3)$$

$$= \boxed{5}$$

* substitute in -1 for x

* derivative

OR

$x - 2x^2$

$1 - 4x$

$1 - 4(-1)$

$\boxed{5}$

14.4 mini notes

② derivative of $f(x) = \frac{3}{x}$ at $(1, 3)$

$$\frac{f(x) - f(c)}{x - c} = \frac{\frac{3}{x} - 3}{x - 1} = \frac{\frac{3 - 3x}{x}}{x - 1}$$

$$= \frac{3 - 3x}{x(x-1)}$$

factor →

$$= \frac{-3(x-1)}{x(x-1)}$$

$$= \frac{-3}{x}$$

$$= \frac{-3}{1} = \boxed{-3}$$

* substitute in 1 for x

14.4 mini Notes

(3) * find derivative on calculator *

by hand

derivative of $\cos x \longrightarrow -\sin x$

derivative of $\sin x \longrightarrow \cos x$

Therefore

$x \cos x$

$(1)\cos x + (-\sin x)(x)$

derivative of 1st term times the second term (der of x is 1) + derivative of second term times the 1st term (der of cos is -sin)

$\cos x - x \sin x$

$\cos(\pi/4) - \pi/4 (\sin \pi/4)$

$\frac{\sqrt{2}}{2} - \frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right) = .152$

mini notes 14.4

(4) $-16t^2+80t = s(t)$

a) derivative

$-32t+80$

$-32(1.5)+80 =$ **32 ft/sec**

b) $-16t^2+80t = 0$

$-16t(t-5)=0$

$t=0 \quad t=5$

---

derivative

$-32t+80$

$-32(5)+80 =$ **-80 ft/sec**