Student: Key **Date:**	**Instructor:** Joe Betters **Course:** Pre-Calculus Pre AP (Master Course)	**Assignment:** 14.4 HW 2020 (Adjusted)

1. Find the slope of the tangent line to the graph of $f(x) = 8x + 2$ at the point $(-1, -6)$. Graph f and the tangent line.

Type the slope of the tangent line, m_{tan}, in the box below.

$m_{tan} = \boxed{8}$ (Type an integer or a simplified fraction.)

$$\frac{f(x) - f(c)}{x - c}$$

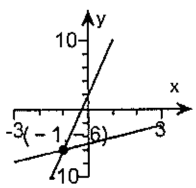
$$\frac{8x + 2 - (-6)}{x - (-1)}$$

$$\frac{8x + 8}{x + 1}$$

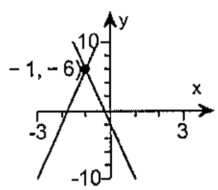
$$\frac{8\cancel{(x+1)}}{\cancel{(x+1)}} = 8$$

Which graph shown below is the correct graph of the original function and the tangent line?

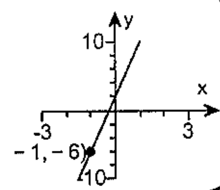
○ A.



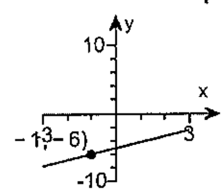
○ B.



○ C.



○ D.



ID: 14.4.9

2. Find the slope of the tangent line to the graph of $f(x) = 3x^2$ at the point (1,3). Graph f and the tangent line.

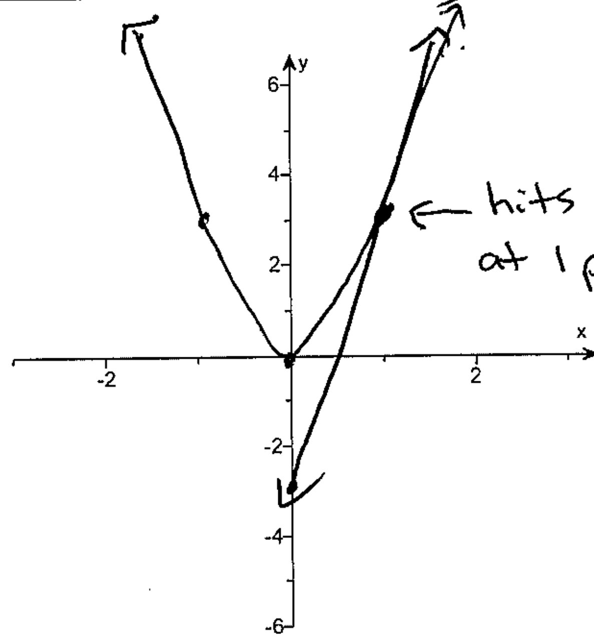
Type the slope of the tangent line, m_{tan}, in the box below.

$m_{tan} = \boxed{6}$ (Enter the value as an integer or reduced fraction.)

Use the graphing tool to graph the function and the tangent line.

$$\frac{f(x) - f(c)}{x - c} = \frac{3x^2 - 3}{x - 1}$$

$$\frac{3(x^2 - 1)}{(x - 1)}$$



← hits exactly at 1 point

ID: 14.4.13

$$\frac{3(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

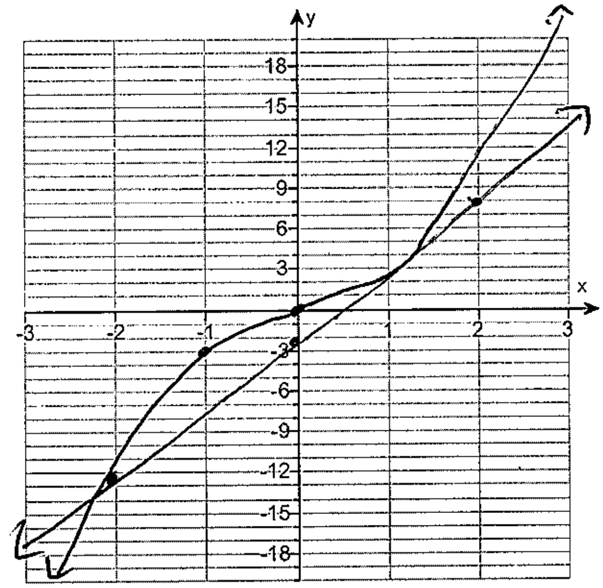
* substitute in 1 for x $\quad 3((1) + 1) = \boxed{6}$

3. Find the slope of the tangent line to the graph of f at the given point. Graph f and the tangent line.

$f(x) = x^3 + 2x$ at (1,3)

The slope of the tangent line to the graph of $f(x) = x^3 + 2x$ at (1,3) is $m_{tan} = $ 5.
(Type an integer or a simplified fraction.)

Use the graphing tool to graph the function and the tangent line.



$$\frac{f(x)-f(c)}{x-c} = \frac{x^3+2x-3}{x-1}$$

* split −3 up so you can factor

$$\frac{(x^3-1)+(2x-2)}{x-1}$$

$$\frac{\cancel{(x-1)}(x^2+x+1)+2\cancel{(x-1)}}{\cancel{(x-1)}}$$

* sub 1 for x

$1^2+1+1+2$
$=5$

~~ID: 14.4.19~~

4. Find the derivative of $f(x) = -7x + 7$ at 3. That is, find $f'(3)$.

$f'(3) = $ −7 (Enter the value as an integer or reduced fraction.)

$f(3) = -14$

$$\frac{f(x)-f(c)}{x-c} = \frac{-7x+7-(-14)}{x-3}$$

$$\frac{-7x+21}{x-3} = \frac{-7\cancel{(x-3)}}{\cancel{x-3}} = -$$

ID: 14.4.21

5. Find the derivative of $f(x) = x^3 - 6x^2 + x$ at 2. That is, find $f'(2)$.

$f'(2) = $ −11 (Enter the value as an integer or reduced fraction.)

* Shortcut alert *

$3x^2-12x+1$ plug in 2 for x

$3(2)^2-12(2)+1$
$=-11$

ID: 14.4.29

6. Find the derivative of the function $f(x) = 7 \sin x + 8$ at 0.

$f'(0) = $ 7

derivative of sin = cos

ID: 14.4.31

7 cos x sub in zero 7 cos 0 = 7

7. Use a graphing utility to find the derivative of $f(x) = \dfrac{-x^3+8}{x^2+6x+3}$ at −4. That is, find $f'(-4)$.

$f'(-4) = $ (Round the answer to six decimal places, if necessary.)

* worked on separate sheet *

ID: 14.4.35

5 * Shortcut $x^{3} - 6x^{2} + x^{1}$

* one less power

$3x^2 - 2(6)x + 1x^0$

8. Use a graphing utility to find the derivative of the function $f(x) = 3x^2 \sin x$ at $\frac{\pi}{4}$.

$f'\left(\frac{\pi}{4}\right) =$ ______

(Round to six decimal places as needed.)

* See attached Sheet

ID: 14.4.39

9. Use a graphing utility to find the derivative of $f(x) = e^x \cos x$ at -7. That is, find $f'(-7)$.

$f'(-7) =$ ______ (Round the answer to three decimal places, if necessary.)

* see attached sheet

ID: 14.4.41

10. **Instantaneous Rate of Change** The volume V of a right circular cylinder of height 6 feet and radius r feet is $V = V(r) = 6\pi r^2$. Find the instantaneous rate of change of the volume with respect to the radius r at $r = 8$.

$V'(8) =$ ______ ft^3/ft

(Leave π as a symbol in your answer. Do not use an approximation for the value of π.)

* See attached Sheet

ID: 14.4.43

11. The volume V of a sphere of radius r feet is $V = V(r) = \frac{4}{3}\pi r^3$. Find the instantaneous rate of change of the volume with respect to the radius r at $r = 6$.

The answer is ______ ft^3/ft.

(Type an exact answer in terms of π.)

* See attached Sheet

ID: 14.4.45

12. In physics it is shown that the height s of a ball thrown straight up with an initial speed of 256 ft/sec from ground level is $s = s(t) = -16t^2 + 256t$, where t is the elapsed time that the ball is in the air.

(a) When does the ball strike the ground? That is, how long is the ball in the air?

16 seconds

$-16t^2+256t=0$ $t(-16t+256)=0$

$t=0$ $t=16$

(b) What is the average speed of the ball from t = 0 to t = 3?

208 ft/sec

$\frac{t(3)-t(0)}{3-0} = \frac{624-0}{3} = 208$

(c) What is the instantaneous speed of the ball at time t?

$s'(t) =$ $-32t+256$

* take derivative $-32t+256$

(d) What is the instantaneous speed of the ball at t = 3?

160 ft/sec

take derivative $-32t+256$ sub in 3 for t

(e) When is the instantaneous speed of the ball equal to zero?

8 seconds

set derivative = 0 $-32t+256=0$ $t=8$

(f) How high is the ball when its instantaneous speed equals zero?

1024 feet

substitute 8 into original

$-16(8)^2+256(8)$

$=1024$

(g) What is the instantaneous speed of the ball when it strikes the ground?

-256 ft/sec

substitute 16 into derivative

$-32(16)+256$

ID: 14.4.47

(7) derivative of fraction using short cut

$$\frac{(\text{derivative of top})(\text{bottom}) - (\text{derivative bottom})(\text{top})}{\text{bottom}^2}$$

$* \ \dfrac{-x^3+8}{x^2+6x+3}$ * use shortcut like on #5

$$\frac{(-3x^2)(x^2+6x+3) - (2x+6)(-x^3+8)}{(x^2+6x+3)^2}$$

* substitute in (-4) for x

$$= \frac{384}{25} = \boxed{15.36}$$

*note

$-x^{3}+8x^{0}$ $x^{2}+6x^{1}+3x^{0}$

$-3x^2+0$ $2x+6+0$

$-3x^2$ derivative $2x+6$ derivative

* subtract exponent by 1

8 derivative of product of terms short cut

(derivative 1st term)(2nd term) + (derivative 2nd term)(1st term)

$(3x^2)(\sin x)$

$6x(\sin x) + (\cos x)(3x^2)$

* substitute in $\pi/4$ for x

$6(\pi/4)(\sin \pi/4) + (\cos \pi/4)(3(\pi/4)^2)$

$= \boxed{4.640699}$

* note

$3x^2$

$6x^1$

$6x$ derivative

$\sin x$

* derivative of $\sin x = \cos x$

* derivative of $\cos x = -\sin x$

$\cos x$ derivative

9 derivative of $e^x = e^x$

derivative of $\cos x = -\sin x$

* use shortcut

(der 1st)(2nd) + (der 2nd)(1st)

$e^x \cos x$

$e^x(\cos x) + (-\sin x)e^x$

* substitute in -7 for x

$= \boxed{.001}$

* note

e^x derivative is e^x

$\cos x$ derivative is $-\sin x$

10 $V = V(r) = 6\pi r^2$

* just take derivative

$(6\pi)(r^2)$

*(der 1st)(2nd) + (der 2nd)(1st)

$0(r^2) + 2r(6\pi)$

$= 12\pi r$

* substitute in 8 for r

$12\pi(8) = \boxed{96\pi}$

(11) $V = V(r) = \frac{4}{3}\pi r^3$

* take derivative using short cut

(der 1st)(2nd) + (der 2nd)(1st)

$\left(\frac{4}{3}\pi\right)\left(r^3\right)$

$0\left(r^3\right) + 3r^2\left(\frac{4\pi}{3}\right)$

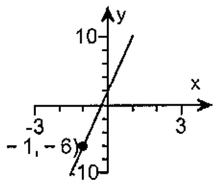
$= 4\pi r^2$

* substitute in 6 for r

$4\pi(6)^2$

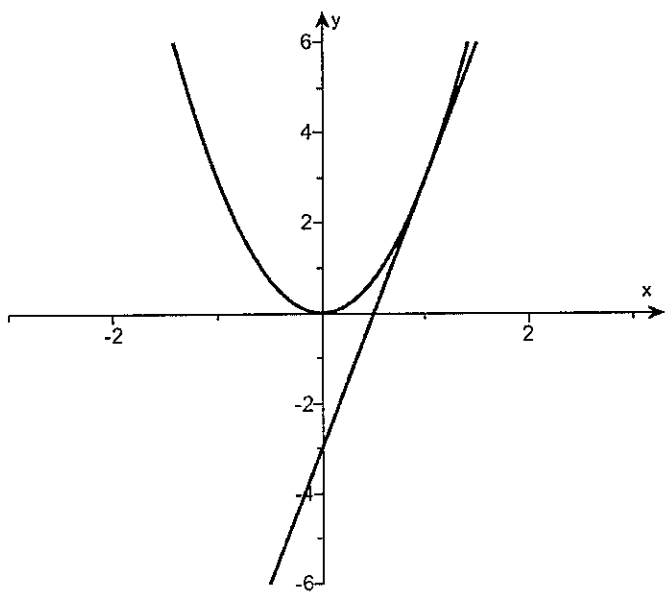
$\boxed{144\pi}$

1. 8

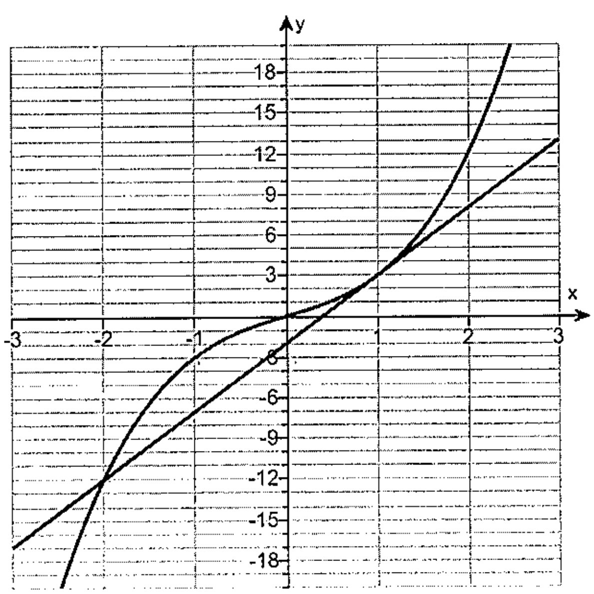


C.

2. 6



3. 5



4. -7

5. -11

6. 7

7. 15.36

8. 4.640699

9. 0.001

10. 96π

11. 144π

12. 16

208

$-32t+256$

160

8

1024

-256