Mini-Lecture 14.3
One-sided Limits; Continuous Functions

Learning Objectives:

1. Find the One-sided Limit of a Function (p. 884)
2. Determine Whether a Function is Continuous (p. 886)

Examples:

1. $f(x)=\begin{cases} \dfrac{x^2-25}{x-5} & \text{if } x\neq 5 \\ 10 & \text{if } x=5 \end{cases}$, find a) $\lim\limits_{x\to 5^-} f(x)$, b) $\lim\limits_{x\to 5^+} f(x)$, c) $\lim\limits_{x\to 5} f(x)$.

Use the definition of continuity to determine whether f is continuous at a.

2. $f(x)=\dfrac{x-6}{x+6}$, $c=-6$

3. $f(x)=\begin{cases} x+2 & \text{if } x<-2 \\ 0 & \text{if } x=-2 \\ x^2-4 & \text{if } x>-2 \end{cases}$; $c=-2$

Determine for what numbers, if any, the given function is discontinuous.

4. $f(x) = \dfrac{x+2}{12x^2+x-6}$

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① $f(x) = \begin{cases} \dfrac{x^2-25}{x-5} & \text{if } x \neq 5 \\ 10 & \text{if } x = 5 \end{cases}$

a) $\lim_{x \to 5^-} f(x) = \dfrac{(x+5)\cancel{(x-5)}}{\cancel{(x-5)}} = x+5$

$5+5$

* approaches 5 from negative side

$*\underline{\underline{4.99999}}+5 \longrightarrow = \boxed{10}$

b) $\lim_{x \to 5^+} f(x) = \dfrac{(x+5)\cancel{(x-5)}}{\cancel{(x-5)}} = x+5$

$5+5$

* approaches 5 from positive side

$*\underline{\underline{5.000001}}+5 \longrightarrow = \boxed{10}$

c) $\lim_{x \to 5} f(x) =$ * since $\lim_{x \to 5^-}$ and $\lim_{x \to 5^+}$

both equals 10,

then the limit $\lim_{x \to 5} = \boxed{10}$

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② $f(x) = \dfrac{x-6}{x+6}$ $\quad c = -6$

$$\lim_{x \to -6^+} \frac{x-6}{x+6} = \frac{-5.99999-6}{-5.99999+6} = \frac{-11.99999}{.00001}$$

$$= \boxed{-\infty}$$

$$\lim_{x \to -6^-} \frac{x-6}{x+6} = \frac{-6.0000001-6}{-6.0000001+6} = \frac{-12.0000001}{-.0000001}$$

$$= \boxed{\infty}$$

at -6, undefined

not continuous

* since $-\infty$ and ∞ are not the same, the limit does not exist at -6

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③ $f(x) = \begin{cases} x+2 & \text{if } x < -2 \\ 0 & \text{if } x = -2 \\ x^2-4 & \text{if } x > -2 \end{cases}$; $c = -2$

$\lim_{x \to -2^-} = 0$

$\lim_{x \to -2^+} = 0$

all approach zero

no gaps

$\boxed{\text{Continuous}}$

* value for every x

④ $f(x) = \dfrac{x+2}{12x^2+x-6}$

* will be discontinuous when denominator is zero

factor $\dfrac{x+2}{(4x+3)(3x-2)}$

discontinuous at $\boxed{-3/4,\ 2/3}$