

Mini-Lecture 14.2
Algebraic Techniques for Finding Limits

Learning Objectives:

1. Find the Limit of a Sum, a Difference, and a Product (p. 878)
2. Find the Limit of a Polynomial (p. 879)
3. Find the Limit of a Power or a Root (p. 880)
4. Find the Limit of a Quotient (p. 881)
5. Find the Limit of an Average Rate of Change (p. 882)

Examples:

Find each limit algebraically.

1. $\lim_{x \rightarrow 3} \frac{x^3 - 6x^2 + 9x}{x^4 - 3x^3 + 2x - 6}$ 2. $\lim_{x \rightarrow -2} \sqrt{10 - 3x}$ 3. $\lim_{x \rightarrow 2} x^3 - 8x^2 + x - 1$

4. Find the limit as x approaches c of the average rate of change of the function from c to x .

$$c = 1; f(x) = \frac{1}{x^3}$$

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$$\textcircled{1} \lim_{x \rightarrow 3} \frac{x^3 - 6x^2 + 9x}{x^4 - 3x^3 + 2x - 6} \leftarrow \begin{array}{l} * \text{denominator} \\ = 0 \text{ if } x=3 \\ \text{so you have} \\ \text{to factor } * \end{array}$$

$$\frac{x(x^2 - 6x + 9)}{x^3(x-3) + 2(x-3)} \leftarrow \begin{array}{l} \text{factor by} \\ \text{grouping} \end{array}$$

$$\frac{x(x-3)(x+3)}{(x^3+2)(x-3)} \leftarrow \text{cancel out}$$

$$\frac{(3)((3)-3)}{(3^3+2)} = \frac{0}{29} = \boxed{0}$$

Substitute
3 in for x
and solve

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$$\textcircled{2} \lim_{x \rightarrow -2} \sqrt{10 - 3x}$$

$$\sqrt{10 - 3(-2)}$$

$$\sqrt{10 + 6} = \sqrt{16} = \boxed{4}$$

* substitute in -2 for x and make sure $\sqrt{\quad}$ is greater than or equal to zero under radical

$$\textcircled{3} \lim_{x \rightarrow 2} x^3 - 8x^2 + x - 1$$

$$(2)^3 - 8(2)^2 + (2) - 1 = \boxed{-23}$$

* no denominator, so just substitute in 2 for x and solve

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④ $C=1$ $f(x) = \frac{1}{x^3}$

rate of change

$$\frac{f(x) - f(\text{limit})}{x - (\text{limit})}$$

$$\frac{\frac{1}{x^3} - \frac{1}{(1)^3}}{x - (1)}$$

value for C

$$\begin{aligned} \frac{\frac{1}{x^3} - 1}{x-1} &= \frac{1-x^3}{x^3(x-1)} = \frac{-(x^3-1)}{(x-1)x^3} \\ &= \frac{-(x-1)(x^2+x+1)}{x^3(x-1)} \\ &= \frac{-((1)^2+(1)+1)}{(1)^3} \\ &= \boxed{-3} \end{aligned}$$

* substitute
 $C=1$ for x
when completely
factored
and reduced